Trading Gains and Productivity: A Törnqvist Approach

Ulrich Kohli[1]
University of Geneva

Abstract

This article looks at alternative Törnqvist measures of a country's trading-gain and terms-of-trade effects, as they have been proposed in the literature starting with the seminal work of Diewert and Morrison (1986), and their link to standard measures of productivity. It strongly argues in favour of using the price of domestic final demand as a deflator when computing real Gross Domestic Income (GDI), and, by the same token, the trading gains and labour productivity measures. It shows that the trading gains then generally consist of two parts, a pure terms-of-trade component and an additional relative-price component, the latter of which can be interpreted as a real-exchange-rate effect. National and international statistical agencies, with the notable exceptions of Statistics Canada and the U.S. Bureau of Economic Analysis, tend to report incomplete trading-gain statistics in that they omit the second component. Consequently the real GDI estimates they publish must be viewed as flawed. Taking trading-gains into account has no direct effect on the measurement of total factor productivity, but it does affect the measures of average and marginal labour productivity when related to real GDI and its deflator. Numerical estimates for Switzerland are reported as an illustration.

It is well known that changes in the terms of trade and the real exchange rate of an open economy can have a significant effect on its welfare. Yet, the impact of such changes on a country's real income — as captured by the so-called trading gains — have long been rather neglected by the traditional measures of the national accounts.[2] Admittedly, trading gains tend to be much smaller than productivity advances, but they can nonetheless be significant. Moreover, the two types of gains may be intertwined. The purpose of this article is to document these effects using superlative price and quantity indices.

Among the statistical agencies, the

1 Emeritus Professor of Economics, University of Geneva. Over the years, I have greatly benefited from discussions with numerous colleagues on the issues covered in this article. At the risk of forgetting many names, I would like to especially thank (more or less in chronological order) W. Erwin Diewert, Alan D. Woodland, Elie Appelbaum, Ronald W. Jones, Henryk Kierzkowski, Jean-Christian Lambelet, Jaime de Melo, Kevin J. Fox, and Marshall B. Reinsdorf. I am also very grateful to the editors and to three anonymous referees for helpful comments and suggestions. Email: Ulrich.Kohli@hotmail.com, Ulrich.Kohli@unige.ch.

2 See Geary (1961) for an early and lucid exposition of the need for such a concept. The term "trading gain" seems to have been coined by Burge and Geary (1957); see Neary (1997).

U.S. Bureau of Economic Analysis (BEA) stands out for long having been publishing series of *command-basis* real Gross Domestic Product (GDP), generally interpreted in the literature as real Gross Domestic Income (GDI) (Denison, 1981). Originally, the BEA's approach was to deflate the trade account by the price of imports, as opposed to deflating exports and imports by their own respective prices as it is usually done when computing real GDP. The difference between the two measures was interpreted as the trading gains, or losses.[3]

Before proceeding, it would seem useful to try to define the concept of trading gain more precisely, rather than simply referring to the statistical approach originally used by the BEA. Thus, one might define the *trading gain* (or *loss*) as the extra real domestic income that a country earns (or loses) simply as the result of changes in the relative prices relevant for its international trade. As it will be shown, these relative prices generally involve at least three prices: the prices of imports, exports, and domestic final goods.

It is noteworthy that most statistical agencies do not define real GDI as nominal GDI (equal to nominal GDP by the national accounts identity) deflated by an appropriate price index. Instead, real GDI is still generally computed as real GDP plus the trading gains, however defined.[4] This is all the more surprising given that, for many purposes, real GDI is just as important a macroeconomic concept as real GDP. Real income is essential in explaining aggregate demand and savings, plays a leading role in many fields of economics, like public finance and monetary economics, and it is a better welfare indicator than real GDP. Real income and trading gains also play an important role in many models of international economics, including the modelling of internal and external balance (Salter, 1959; Corden, 1960). Nonetheless, the estimation of real GDI is generally relegated to a side issue and is subjected to the vagaries of the measurement of the trading gains.

Both the *System of National Accounts* (SNA) *2008* and Eurostat's *European System of Accounts* (ESA) *2010* now do recommend that trading gains be treated as an integral part of the SNA.[5] They leave the choice of the price deflator to the individual countries, however, simply suggesting one of the following: the price of imports, the price of exports, an average of the two, a general price index like the consumer price index, or a price index for gross domestic final expenditures. In recent years, many countries have thus begun to publish data on trading gains, mostly using the price of imports as the deflator of the trade account. Some countries opted for a domestic price index instead. Thus, Switzer-

3 The *trading gains* are measured relative to a reference period; this is not to be confused with the gains from trade, which traditionally refer to a (hypothetical) closed-economy situation.

4 Thus, the *Export and Import Price Manual* defines real GDI as: "A real income measure defined as the volume of GDP plus the trading gain or loss resulting from changes in the terms of trade," International Monetary Fund (2009b:619).

5 See International Monetary Fund (2009a:317), and European Commission (2013:302). The Stiglitz Commission also recommended that trading gains be taken into account (Stiglitz, Sen, and Fitoussi, 2009:95); Hartwick (2020) also discussed this issue in his very extensive review of national accounting.

land started publishing trading-gain statistics using the gross domestic final expenditure price index as deflator in July 2007.[6] Canada did likewise in December 2008, and the United States followed suit in July 2010 (Statistics Canada, 2016, Chapter 7:28).

While the BEA originally used fixed-weight Laspeyres quantity indices when computing trading gains, it started using chained Fisher price and quantity indices in 1996.[7] Statistics Canada did the same in 2001, but as of today most other countries, including Switzerland, still use the Laspeyres quantity aggregation, albeit in chain form.

The focus in this article is on chained Törnqvist — rather than Fisher — indices. This choice is motivated by their ease of computation and exposition, plus the fact that the Translog functional form, for which Törnqvist indices are exact, can be estimated relatively easily (Christensen, Jorgenson and Lau, 1973; Diewert, 1974). Moreover, there is no known functional form for a GDP function for which Fisher indices are exact, except under some rather restrictive restrictions such as global separability between domestic factor services and output (including import) quantities (Kohli, 1993; Kohli, 2004a; footnote 21). In any case, it is widely acknowledged that the numerical differences between these two superlative indices are typically very small (Diewert, 1976).

We will show that there are compelling arguments in favour of using the price index for gross domestic final expenditures as a deflator when computing real GDI and the trading gains. Moreover, except for the unlikely situation when trade is balanced, the trading gains really consist of two elements, a pure terms-of-trade effect and a further relative-price effect that can be interpreted in some cases as a real-exchange-rate effect. Most statistical agencies only report the first component, which means that their so-called trading-gain estimates are incomplete, and thus misnamed, and, furthermore, that their measures of real GDI are flawed.

The Diewert and Morrison Approach to Terms-of-trade Effects

Our starting point is the seminal *Economic Journal* article by Erwin Diewert and Catherine Morrison (Diewert and Morrison, 1986). They use the GDP-function approach to modelling imports and exports, which treats traded goods as middle products.[8] This approach recognizes the fact that most imports are made up of raw materials and intermediate products, and even most so-called finished products must still go through a number of domestic transformations (such as transportation, insurance, unloading, storage, wholesaling, and retailing), where they re-

6 See Swiss National Bank (2007:page IV); these series were extended back to 1990; thanks are due to Michel Peytrignet, former Head of Economic Affairs, and Christoph Menzel, former Head of Statistics, for their role in having these series published.

7 See Landefeld and Parker (1997); also see Reinsdorf (2010) for a very thorough and detailed analysis of trading gains in the context of chained Fisher indices; Reinsdorf makes a very strong case in favour of the use of the gross domestic final expenditures price index as deflator.

8 See Kohli (1978, 1991) and Woodland (1982); the term middle product was coined by Sanyal and Jones (1982).

ceive domestic value added before eventually reaching final demand. The same holds true for exports that can be viewed as intermediate inputs to the foreign technology. In essence, nearly all international trade takes place during production, rather than after.

Define the country's production possibilities set (Θ_t) as the set of all feasible input and output combinations at time t. Let $p_{i,t}$ be the price of output i at time t and $q_{i,t}$ its quantity, and let $x_{j,t}$ and $w_{j,t}$ be the quantity and the rental price of domestic primary factor j; the corresponding vectors are denoted by $p_t \equiv \lfloor p_{i,t} \rfloor, q_t \equiv \lfloor q_{i,t} \rfloor, x_t \equiv \lfloor x_{j,t} \rfloor, w_t \equiv \lfloor w_{j,t} \rfloor$. For illustrative purposes, and since we are mostly interested in imports and exports, we will assume just three variable quantities: exports (X), imports (M, treated as a negative output), and domestic final expenditures (N, an aggregate of private consumption, government consumption, and investment). Note that the domestic final good is clearly distinct from imports and exports, and it can be therefore interpreted as a non-traded good. Production involves two domestic factors, labour (L) and capital (K), both in fixed supplies at any point in time. Assuming that Θ_t is a convex cone and that production is competitive and profit maximizing, the technology can be represented by a GDP function defined as follows:[9]

$$\pi_t = \pi(p_t, x_t, t) \equiv max_q \begin{Bmatrix} p_{N,t}q_N + p_{X,t}q_X \\ -p_{M,t}q_M : (q, x_t) \in \Theta_t, \end{Bmatrix} \quad (1)$$

where π_t is nominal GDP at time t. This GDP function is linearly homogeneous in prices by definition. Moreover, the assumption that Θ_t is a cone implies constant returns to scale, i.e. linear homogeneity in domestic input quantities. It can conveniently be implemented using the Translog functional form; it is well known that this function can provide a second-order approximation to an arbitrary GDP function such as (1).

Let $\Pi_{t,t-1} \equiv \pi_t/\pi_{t-1}$ be the growth factor of nominal GDP. Diewert and Morrison show that it can be expressed as:

$$\Pi_{t,t-1} = P_{Y,t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1} \quad (2)$$

where

$$P_{Y,t,t-1} \equiv \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{\bar{s}_{N,t}} \quad (3)$$

is a Törnqvist index of the prices of outputs (including imports, treated as a negative output), and

$$X_{t,t-1} \equiv \left(\frac{x_{L,t}}{x_{L,t-1}}\right)^{\bar{\sigma}_{L,t}} \left(\frac{x_{K,t}}{x_{K,t-1}}\right)^{\bar{\sigma}_{K,t}} \quad (4)$$

is a Törnqvist index of the quantities of the fixed domestic factors; $s_{i,t}(i = N, X, M)$ and $\sigma_{j,t}(j = K, L)$ are the nominal GDP shares of output i and input j at time t, respectively, with $s_{X,t} - s_{M,t} + s_{N,t} = 1$ and $\sigma_{L,t} + \sigma_{K,t} = 1$; $\bar{s}_{i,t} \equiv \frac{1}{2}(s_{i,t-1} + s_{i,t})$ and $\bar{\sigma}_{j,t} \equiv \frac{1}{2}(\sigma_{j,t-1} + \sigma_{j,t})$ denote the av-

9 See Diewert (1974), Kohli (1978, 1991), and Woodland (1982) for the properties of GDP functions.

erage share of output i and input j over consecutive periods. Diewert and Morrison demonstrate that both of these indices are exact if the underlying GDP function is indeed Translog. $R_{t,t-1}$, finally, is a measure of total factor productivity (TFP) growth and it is obtained as a residual:[10]

$$R_{t,t-1} \equiv \Pi_{t,t-1}/(P_{Y,t,t-1} \cdot X_{t,t-1}). \quad (5)$$

Considering expression (2), both $X_{t,t-1}$ and $R_{t,t-1}$ are real growth factors and their product yields the real-GDP growth factor, $Y_{t,t-1} \equiv y_t/y_{t-1}$:[11]

$$Y_{t,t-1} \equiv \Pi_{t,t-1}/P_{Y,t,t-1} = X_{t,t-1} \cdot R_{t,t-1}. \quad (6)$$

This expression shows that the two sources of economic growth are the increases in factor endowments, as captured by $X_{t,t-1}$, and increases in productivity, as measured by $R_{t,t-1}$.

Diewert and Morrison convincingly argue, however, that $P_{Y,t,t-1}$ in (2) does also contain a real element, namely the impact of changes in the terms of trade. An improvement in the terms of trade is similar to a technological progress, in that it allows a country to obtain more for less, so to speak. It is as if a country's exports were transformed into its imports by the rest of the world. The fact that this transformation takes place abroad rather than within the country is not relevant from a strictly economic viewpoint, and if this transformation technology becomes more (or less) productive over time, it has very real consequences on the country's income. Diewert and Morrison therefore seek to exclude this real component from $P_{Y,t,t-1}$ and they define the following terms-of-trade effect, $DMA_{t,t-1}$:

$$DMA_{t,t-1} \equiv \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}}. \quad (7)$$

This term captures the impact on nominal GDP of changes in import and export prices. The decomposition of nominal GDP growth then becomes:

$$\Pi_{t,t-1} = DMA_{t,t-1} \cdot DMB_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1} \quad (8)$$

where

$$DMB_{t,t-1} \equiv \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{1-(\bar{s}_{X,t}-\bar{s}_{M,t})} = \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{\bar{s}_{N,t}} \quad (9)$$

measures the nominal-GDP effect of changes in the price of the domestic final good.

Decomposition (8) has been used in a number of empirical studies to explain the growth of nominal GDP (Fox and Kohli, 1998; Fox, Kohli, and Warren, 2002; Kohli, 1990, 2002). One drawback of this approach, however, is that $DMA_{t,t-1}$, which is supposed to measure a real effect, is not homogeneous of degree zero in prices, unless trade happens to be balanced over consecutive periods (Kohli, 2003, footnote 25; Kohli, 2004a, footnote 19). In

10 It is possible to calculate $R_{t,t-1}$ exactly if the parameters of the Translog GDP function are known (Kohli, 1990).

11 This index of real GDP thus has the implicit Törnqvist form (Kohli, 2004b).

other words, an equiproportionate change in import and export prices would generally lead $DMA_{t,t-1}$ to register a change, even though the terms of trade clearly would not have varied, thus suggesting that $DMA_{t,t-1}$ is not a measure of a pure terms-of-trade effect. By the same token, if trade is not balanced (i.e. if $\bar{s}_{N,t} \neq 1$), the price term $DMB_{t,t-1}$ is not linearly homogeneous in current prices as one would expect it to be: it must therefore still contain a real element. This qualification may not be important for a majority of countries whose trade is close to being balanced, but there are also many countries that do not satisfy this requirement.

Terms-of-trade and Real-exchange-rate Effects

To address the problem of the non-zero price homogeneity of the terms-of-trade effect, a number of different approaches have been proposed in the literature. The idea behind all of them is to recognize, as suggested in the introduction, that, unless trade happens to be balanced, the trading gains do not merely depend on the prices of imports and exports, but also on a third price, the price of the domestic final good. These three prices can be characterized by two price ratios, one of which being the terms of trade, whereas the second ratio can be defined in different ways, each time relative to $p_{N,t}$. It is important, though, that both ratios be taken into account when deriving the trading gains, not only for the estimate of these to be complete, but also to ensure that the decomposition of nominal GDP be linearly homogeneous in prices. Whether trade is balanced or not, $p_{N,t}$, the price of the domestic final good then emerges naturally as the appropriate deflator for real GDI. Some of these approaches are discussed in the on-line Appendix.[12] In what follows, we will use the approach of Kohli (2006a, 2006b, 2007), which is the most appealing from a trade-theoretic viewpoint.[13]

We begin by defining the terms of trade (h_t) as the ratio of export prices to import prices:

$$h_t \equiv \frac{p_{X,t}}{p_{M,t}}. \qquad (10)$$

We next define the price of traded goods ($p_{T,t}$) as a weighted geometric mean of the price of exports and imports:

$$p_{T,t} \equiv p_{X,t}^{\lambda} \cdot p_{M,t}^{1-\lambda} \quad 0 \leq \lambda \leq 1. \qquad (11)$$

The weight on the price of exports (λ) could be set to ½ in analogy to one of the options mentioned in the introduction. Alternatively, it could be set to the share of exports in total trade in the first period, or the mean share over the sample.

Finally, we define the real exchange rate (e_t) as the price of traded relative to the price of non-traded goods:[14]

12 The on-line Appendix can be found on the Centre for the Study of Living Standards Website: http://www.csls.ca/ipm/42/IPM_42_Kohli_Appendix.pdf

13 Also on this issue, see Kohli and Natal (2014), Macdonald (2010, 2020), Macdonald and Ripsoli (2016), and Reinsdorf (2010).

14 This measure of the real exchange rate is also known in the literature as the Salter (1959) ratio; on this topic, also see Corden (1992).

$$e_t \equiv \frac{p_{T,t}}{p_{N,t}} = \frac{p_{X,t}^{\lambda} \cdot p_{M,t}^{1-\lambda}}{p_{N,t}}. \quad (12)$$

An increase in e_t means, ceteris paribus, a real depreciation of the home currency as internationally traded goods become relative more expensive.

Let $P_{N,t,t-1} \equiv p_{N,t}/p_{N,t-1}$; it can be shown that the following exact decomposition holds if the underlying GDP function has the Translog form:[15]

$$\Pi_{t,t-1} = P_{N,t,t-1} \cdot H_{t,t-1} \cdot E_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1}, \quad (13)$$

where

$$H_{t,t-1} \equiv \left(\frac{h_t}{h_{t-1}}\right)^{(1-\lambda)\bar{s}_{X,t}+\lambda\bar{s}_{M,t}} = \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{(1-\lambda)\bar{s}_{X,t}+\lambda\bar{s}_{M,t}} \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-(1-\lambda)\bar{s}_{X,t}-\lambda\bar{s}_{M,t}} \quad (14)$$

measures the terms-of-trade effect, and

$$E_{t,t-1} \equiv \left(\frac{e_t}{e_{t-1}}\right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})} = \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\lambda(\bar{s}_{X,t}-\bar{s}_{M,t})} \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{(1-\lambda)(\bar{s}_{X,t}-\bar{s}_{M,t})} \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \quad (15)$$

is the real-exchange-rate effect. Note that the welfare effect of a real depreciation of the home currency (an increase in e_t) depends on the position of the trade account as export revenues and the cost of imports both increase: the net effect is positive if the country is in a surplus position, negative otherwise.

Taken together, these two effects capture the complete trading gains as given by factor $G_{t,t-1}$:[16]

$$G_{t,t-1} \equiv H_{t,t-1} \cdot E_{t,t-1} = \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})}. \quad (16)$$

This approach only differs from the one of Kohli (2003, 2004a) discussed in the online Appendix by the decomposition of the trading gains between a terms-of-trade effect and a relative-price effect. The defining advantage of the approach encapsulated by (16) is that the residual, relative-price effect ($E_{t,t-1}$) has a clear economic interpretation, namely that it is a real-exchange-rate effect.

As recommended by the SNA, we can define real GDI (denoted by z_t with $Z_{t,t-1} \equiv z_t/z_{t-1}$) as real GDP augmented by the trading gains; in terms of growth factors:

15 For a proof, see Kohli (2006a, 2007).

16 Reinsdorf (2010) obtains a similar result in the context of the Fisher aggregation.

$$Z_{t,t-1} \equiv Y_{t,t-1} \cdot G_{t,t-1} = G_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1}. \tag{17}$$

Making use of (13), we then find that $p_{N,t}$ can be interpreted as the real GDI price deflator:[17]

$$\Pi_{t,t-1}/Z_{t,t-1} = \Pi_{t,t-1}/(G_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1})$$
$$= P_{N,t,t-1}. \tag{18}$$

This makes considerable sense since the ultimate objective of domestic income is precisely to purchase domestic goods, at price $p_{N,t}$. Moreover, this shows that it is just as easy to compute the full trading gains as the ratio of two price indices:

$$G_{t,t-1} = Z_{t,t-1}/Y_{t,t-1} = P_{Y,t,t-1}/P_{N,t,t-1}, \tag{19}$$

and real GDI can then be obtained simply by deflating nominal GDP (i.e. nominal GDI by the national accounts identity) by $p_{N,t}$.

The Modified Diewert and Morrison Effect

By comparing (7) with (16), the full meaning of the non-zero homogeneity of $DMA_{t,t-1}$ becomes clear: an equiproportionate change in the prices of imports and exports, other things equal, does have a real effect if trade is unbalanced, not because the terms of trade have changed (they have not), but because, by keeping domestic prices constant, it implies a change in the relative prices of traded and nontraded goods. In that case, $DMA_{t,t-1}$ measures a real-exchange-rate effect, not a terms-of-trade effect. $DMA_{t,t-1}$ can best be described as measuring the contribution of changes in import and export prices to the growth in nominal GDP (and nominal GDI). It is only when trade is balanced that $DMA_{t,t-1}$ gives an accurate measure of the terms-of-trade effect, and indeed of the trading gains, the real exchange-rate effect then being nil.

In later work, Diewert and Lawrence (2006) have rewritten expression (7) in terms of relative prices, in which case the zero homogeneity in prices is achieved. In doing so, they used consumption goods as the numeraire, but in order not to depart unnecessarily from the framework used so far, one can opt for the full set of domestic final purchases instead. The modified Diewert and Morrison term ($DMA'_{t,t-1}$) is therefore as follows:

$$DMA'_{t,t-1} \equiv \left(\frac{p_{X,t}/p_{N,t}}{p_{X,t-1}/p_{N,t-1}}\right)^{\bar{s}_{X,t}} \cdot \left(\frac{p_{M,t}/p_{N,t}}{p_{M,t-1}/p_{N,t-1}}\right)^{-\bar{s}_{M,t}}$$
$$= \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})}. \tag{20}$$

Comparing (20) with (16), it appears immediately that $DMA'_{t,t-1} = G_{t,t-1}$, and hence $DMB'_{t,t-1}$, the price term accordingly adjusted, becomes equal to $P_{N,t,t-1}$. That is, the modified Diewert and Morrison term is not a measure of the terms-of-

17 Thus, if $P_{N,t,t-1}$ is computed as a Törnqvist price index, real GDI has the implicit Törnqvist form, just like real GDP; see footnote 11.

trade effect, but of the full trading gains instead. The modified Diewert and Morrison trading-gain measure can then easily be decomposed into a pure terms-of-trade effect and a real exchange-rate effect with the help of (14) and (15).

Note that if one had used the price of consumption as the numeraire as recommended by Diewert and Lawrence (2006), (20) could still be interpreted as a trading-gain index, but (15), the relative-price-effect, could no longer be viewed as a real-exchange-rate effect since changes in the prices of the other nontraded goods (investment and government purchases) would not be caught by (20): they would instead directly affect real income then defined as nominal GDP deflated by the price of consumption goods.

Trading Gains and Productivity

As suggested in the introduction, trading gains and productivity advances are of a similar breed since they both lead to increases in real income for given endowments of primary factors. Moreover, trading gains may affect the measurement of productivity, depending on the definition of productivity that is retained.

One favoured measure of productivity has already been referred to, namely total factor productivity (TFP) as captured by Törnqvist index $R_{t,t-1}$. Identifying the trading gains and adding them to real GDP to obtain real GDI has no impact on the measures of nominal and real GDP. Changes in the prices of exports, imports, and domestic goods are already fully taken into account when computing nominal GDP and its price. Expression (6) remains valid and the measure of TFP is therefore unaffected. For a given change in the endowment of domestic factors as given by $X_{t,t-1}$ if properly measured, $R_{t,t-1}$ is fully determined and thus independent of $H_{t,t-1}$ and $E_{t,t-1}$.[18] The trading gains simply are a benefit in addition to increases in TFP.

More generally, it is noteworthy that if the Törnqvist aggregation is exact for the underlying function, and assuming perfect competition and optimization, a change in any output price, holding technology and factor endowments constant, has no impact on real GDP since it has exactly the same relative effect on nominal GDP and on its price. Put in another way, using a language familiar to trade economists, a change in output (including import) prices will lead to a movement along the production possibilities frontier, but real GDP, adequately measured, is constant along that line.[19] This is not to say that, for given factor endowments and a given technology, a change in the terms of trade or the real exchange rate cannot affect total factor productivity. Quite the contrary: a change in h_t or e_t is likely to have an impact on relative factor rental prices and hence on their income shares, thereby affecting the measure of $X_{t,t-1}$, and, by the same token, the measure of $R_{t,t-1}$ obtained as a resid-

18 Kehoe and Ruhl (2008) reach the same conclusion with a set of different models.

19 Technically speaking, it will be a surface in a three-dimensional space rather than just a line since we are considering three variable quantities.

ual, real GDP remaining unchanged. This, however, is a matter of economic analysis, not an accounting issue. At any point in time, for a given set of output prices, factor endowments and technology, the measure of $R_{t,t-1}$ is independent of whether or not the trading gains have actually been measured and taken into account.

We next consider a second, very common measure of productivity: the average productivity of labour, i.e. the real value added per unit of labour. In fact, we will consider two such measures, one with respect to real GDP and the other with respect to real GDI. We do, however, have a strong preference for the latter given that international trade takes place overwhelmingly in middle products, and thus occurs during the production process rather than afterwards. As such, we view it as problematic to treat trading gains as an afterthought. The singling out of labour might also need a justification. One might think that there is no reason to impute the trading gains to domestic labour since they were obtained from abroad. However, the same could be said when it comes to the production gains resulting from the availability of more advanced equipment, perhaps even imported from abroad. In both cases, though, labour is involved in some way, and it is a convenient shortcut to relate the overall performance of the economy to the work effort: labour is then used as a metric so to speak.[20]

Nonetheless, it might seem a bit far-fetched to include the trading gains in any measure of productivity. One could assert that productivity is a concept intimately linked to the production process and thus to GDP, whereas trading gains are more of an income concept. An improvement in the terms of trade, for example a drop in the price of oil in the case of an oil-importing nation may be the result of pure luck: domestic production factors are without merit in this development and should not be able to claim an improvement in productivity, even though on average their real income will unambiguously increase. This is quite true, but similar situations can occur in a closed economy. Exceptionally poor weather can have a detrimental effect on agricultural production, and hence on measured average labour productivity, without any fault of the farmers who may have been just as hard working as ever. As for the drop in oil prices, it could also result from the completion of a new trans-border pipeline that gives access to a cheaper foreign supplier. As such, it would be difficult to argue that this improvement in the terms of trade is not related to production activities at home and abroad.

Terms-of-trade movements are often viewed as being temporary and likely to self-correct over time, whereas productivity gains due to improving technology are unlikely to be reversed. Admittedly, resource-exporting countries often face volatile terms of trade. Nonetheless, the price cycles may extend over many

20 The measurement of productivity by the real value added per unit of labour is nonetheless often criticized, precisely because it focuses exclusively on one factor of production, namely labour. The wide acceptance of this concept probably has to do in parts because of its early adoption by the Organisation for European Economic Co-operation (OEEC, the ancestor of the OECD) in 1949 under the influence of Jean Fourastié; see Boulat (2006:97).

years and the reversal to mean is not guaranteed. Exporters of industrialized goods are probably less exposed to such volatility. As we shall see below, Switzerland's terms of trade have trended mostly upwards over a 50-year period, while the real exchange rate trended downwards, revealing a steady real appreciation of the currency. Besides, closed economies are not immune either to random, temporary productivity shocks that may be caused by weather, health, social, or political disturbances: standard productivity measures therefore also have to contend with this type of volatility.

More generally, better terms of trade can be the result of a research activity (e.g. market prospection) or of a marketing effort. In a globalized world, firms are constantly searching for new suppliers and additional customers abroad. To the extent that significant quantities of domestic labour and capital are diverted from domestic production to such activities, average labour productivity (and TFP) could be underestimated. Improvement in the terms of trade could also reflect a refinement in the quality of exports that is not fully reflected by the export price and quantity indices. This could also lead to an underestimation of real GDP per unit of labour. Taking the trading gains into account might help to correct for these types of biases.

Better terms of trade can also result from technological advances made abroad. In that sense, the home country may appear to be free-riding on an effort made elsewhere. Note that such a technological advance could also have been made by the foreign subsidiary of a domestic firm and thus have been initiated in the home country. In any case, there is little doubt that globalization and international trade have led to massive transfers of technology and have favoured the international dissemination of productivity gains. For instance, countries throughout the world have greatly benefited from being able to import better and better hi-tech products manufactured in only a handful of countries at ever-lower prices. It is a two-way street, though: while the home country can largely benefit from technological advances made abroad, the rest of the world can also take advantage of the progress made at home.

As already stressed, almost all trade takes place during production, rather than after. In our view the "trade technology," which "transforms" exports into imports, should therefore be treated as an essential element of the country's all-embracing technology. Whether components are transformed into others through a physical process, a chemical reaction, or trade, at home or abroad, should not really matter much to economists. Because it may be difficult in many situations to clearly label what is capital deepening, what is technological progress, what is human capital enhancement, and what are pure trading gains, the line between these concepts tends to be blurred in an integrated world. Given the risk that as a result of measurement errors one development may be wrongly imputed to one or another growth factor speaks in favour of considering all of them jointly. Moreover, the reason why economists are interested in productivity is ultimately that it is income enhancing, and it therefore makes sense to take account of all sources of gains, whether domestic or foreign.

Nonetheless, as mentioned earlier, we will also consider the average labour productivity relative to GDP in what follows; as we shall see, the difference between this "closed-economy" measure and the "open-economy" measure we favour is fully accounted for by the trading gains.

We thus begin by defining $a_{Z,t} \equiv z_t / x_{L,t}$ as real GDI per unit of labour, or, in terms of growth factors:

$$A_{Z,t,t-1} \equiv \frac{Z_{t,t-1}}{X_{L,t,t-1}}, \tag{21}$$

with $A_{Z,t,t-1} \equiv a_{Z,t}/a_{Z,t-1}$ and $X_{L,t,t-1} \equiv x_{L,t}/x_{L,t-1}$. It follows from (17) that this can be expressed as:

$$A_{Z,t,t-1} \equiv (G_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1}) \cdot X_{L,t,t-1}^{-1}. \tag{22}$$

Making use of (4), we find that:

$$\begin{aligned} X_{t,t-1} \cdot X_{L,t,t-1}^{-1} &= \left(\frac{x_{K,t}}{x_{L,t}}\right)^{\bar{\sigma}_{K,t}} \left(\frac{x_{L,t}}{x_{L,t-1}}\right)^{\bar{\sigma}_{L,t}-1} \\ &= \left(\frac{x_{K,t}/x_{L,t}}{x_{K,t-1}/x_{L,t-1}}\right)^{\bar{\sigma}_{K,t}} \\ &= \left(\frac{k_t}{k_{t-1}}\right)^{\bar{\sigma}_{K,t}} \\ &\equiv \ K_{t,t-1}, \end{aligned} \tag{23}$$

with $k_t \equiv x_{K,t}/x_{L,t}$ the capital/labour ratio, and $K_{t,t-1}$ the contribution of capital-intensity changes to economic growth. We thus obtain the following complete Törnqvist decomposition of the growth in this "globalized" version of domestic average labour productivity:

$$\begin{aligned} A_{Z,t,t-1} &= G_{t,t-1} \cdot K_{t,t-1} \cdot R_{t,t-1} \\ &= H_{t,t-1} \cdot E_{t,t-1} \cdot K_{t,t-1} \cdot R_{t,t-1}. \end{aligned} \tag{24}$$

This decomposition is exact if the underlying real GDI function is indeed Translog. Admittedly the last two components are likely to dominate the terms-of-trade and the real-exchange-rate effects, but, in our opinion, the trading gains need nonetheless to be considered to obtain a complete assessment of the change in average labour productivity in the open economy.

Note that it follows from (6) and (23) that the product of the last two components yields the growth in the average labour productivity defined with respect to real GDP, $A_{Y,t,t-1}$, or put another way, the average productivity of labour in a closed-economy setting (Kohli, 2005b):

$$\begin{aligned} A_{Y,t,t-1} &\equiv \frac{Y_{t,t-1}}{X_{L,t,t-1}} \\ &= X_{t,t-1} \cdot R_{t,t-1} \cdot X_{L,t,t-1}^{-1} \\ &= K_{t,t-1} \cdot R_{t,t-1}. \end{aligned} \tag{25}$$

Thus, the only difference between this measure and the one we recommend is the exclusion here of the trading gains.

Yet another important indicator of productivity is the marginal product of labour. As far as workers are concerned, their marginal product is undoubtedly of more interest to them than their average product since the former is directly related to their purchasing power. In the Cobb-

Douglas case, the marginal product of labour is proportional to its average product, but this is generally not true in the case of higher-order functional forms such as the Translog. Under perfect competition and optimization, the marginal product of labour can readily be observed as the real wage rate, $u_{L,t} \equiv w_{L,t}/p_{N,t}$, i.e. the nominal wage deflated by the price of domestic final goods, the GDI price deflator. Note that the nominal wage is an income concept and it therefore would make little sense to use the price of GDP as given by (3) to deflate nominal wages. Domestic residents buy domestic final goods, they do not purchase imports or exports. Thus, in view of (19), the trading gains are automatically taken into account in the definition of the real wage, and the question of whether or not the trading gains should be included in this indicator of productivity is a non-issue.

Recall now that $\sigma_{L,t} \equiv (x_{L,t}w_{L,t})/\pi_t = (x_{L,t}w_{L,t})/(z_t p_{N,t})$; it therefore follows that $u_{L,t} = a_{Z,t} \cdot \sigma_{L,t}$ or, in terms of growth factors:

$$U_{L,t,t-1} = A_{Z,t,t-1} \cdot S_{L,t,t-1}, \qquad (26)$$

where

$$U_{L,t,t-1} \equiv u_{L,t}/u_{L,t-1} \qquad (27)$$

and

$$S_{L,t,t-1} \equiv \sigma_{L,t}/\sigma_{L,t-1}. \qquad (28)$$

Together with (25), this enables us to obtain a complete decomposition of the growth of the marginal product of labour:

$$U_{L,t,t-1} = S_{L,t,t-1} \cdot H_{t,t-1} \cdot E_{t,t-1} \cdot K_{t,t-1} \cdot R_{t,t-1}. \qquad (29)$$

This expression is very handy since each one of its terms can be measured with observed data exclusively. It also shows that, although TFP and capital deepening are almost certainly the main drivers of the growth in the marginal productivity of labour, terms-of-trade and real-exchange-rate effects again cannot be ignored for the decomposition to be complete.

Decomposition (29) is essentially an accounting identity that should hold at any point in time for a given set of output prices, factor endowments, and technology. It is silent, however, as to the economic forces that cause the changes that are being measured. One must recall that all the components of (29) are endogenous to the extent that they all depend on input and output shares. This is of course most obvious for $S_{L,t,t-1}$, unless the underlying technology is Cobb-Douglas, in which case $S_{L,t,t-1} = 1$. The question of how the ratio of the marginal to the average product of labour would change as the result of hypothetical changes in the terms of trade, the real exchange rate, relative factor endowments, and technological progress is an empirical issue, which cannot be answered without a detailed knowledge of the form of the underlying technology. One key parameter is the Hicksian elasticity of complementarity between labour and capital (ψ_{KL}).[21] If ψ_{KL} is greater than one,

21 In the two-input case, the Hicksian elasticity of complementarity is the inverse of the Allen-Uzawa elasticity of substitution.

an increase in the capital-labour ratio will lead to an increase in the labour share, thus meaning that an increase in capital intensity will raise the marginal product of labour by more than its average product. On the other hand, if technological change is mostly Harrod neutral (i.e. labour-augmenting), the passage of time will tend to have an offsetting effect by reducing the labour share for $\psi_{KL} > 1$. Furthermore, although trading gains lead to increases in real domestic income, it is not certain that both factors of production will benefit equally, if at all. It might indeed be the case that one of the two factors becomes worse off — even though the country as a whole is unambiguously better off — if its own income share decreases sufficiently.[22] The sign and the size of the impact of changes in the terms of trade and the real exchange rate on the marginal product of labour depend on the so-called Stolper-Samuelson elasticities, which, in turn, are functions of the parameters of the underlying technology (Kohli, 2010).

Numerical results for Switzerland

Switzerland has, at times, enjoyed very strong improvements in its terms of trade. Given the relatively large size of its foreign trade sector, one would expect this development to have made a significant positive contribution to real GDI. At the same time, Switzerland has experienced a strong real appreciation of its currency and a large trade surplus, which, put together, suggest a negative real-exchange-rate effect. Its total trading gains — or losses — are therefore likely to be nontrivial and it thus seems of interest to have a look at the Swiss data.[23]

Chart 1 shows the path of the Swiss terms of trade (h_t). As it can be seen, they have improved significantly, particularly during the 1980s and 1990s, peaking at nearly 25 per cent by 2003 and falling back somewhat, to 18 per cent by the end of the sample period. The real exchange rate (e_t), on the other hand, fell almost continuously for the first three decades, to reach a level of about 46 per cent below its initial level by 2003, thus revealing a very substantial appreciation, as the relative price of internationally traded goods decreased. The trade balance index, finally, defined here as the ratio of nominal exports to nominal imports, increased steadily starting in the 1980s, thus indicating a growing trade surplus.

We report in Table 1,[24] first column, the Diewert and Morrison terms-of-trade effect, DMA_t, as given by (7), but chained over the entire sample period.[25] Next to

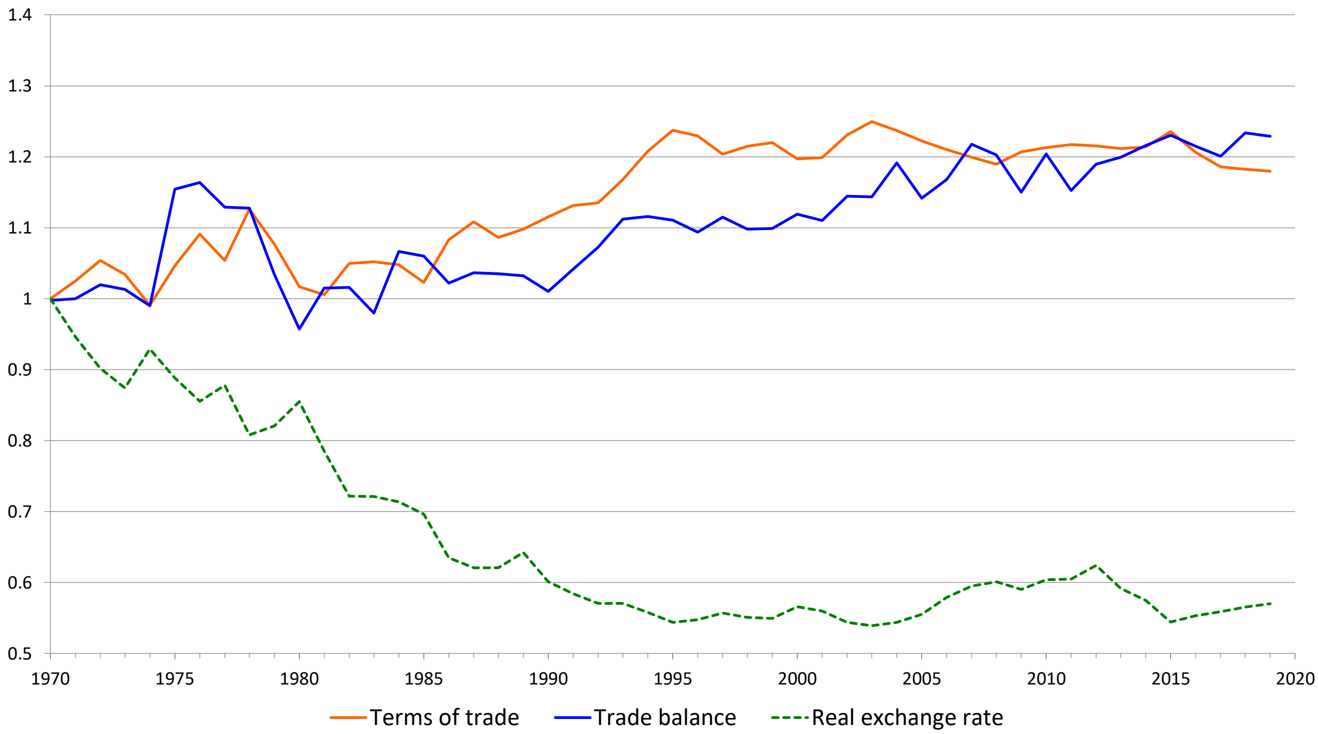
22 This is of course the rule in the well-known two-sector Heckscher-Ohlin-Samuelson model of international trade as the result of the implicit, restrictive non-joint-production hypothesis; see Kohli (1991).

23 These are annual for the period 1970-2019. The output data are taken directly from the OECD data base; the prices and quantities of labour and capital services are derived from the Swiss National Bank and Swiss Federal Statistical Office data bases.

24 See page 16 of this article.

25 Formally, $DMA_t \equiv DMA_{t,t-1} \cdot DMA_{t-1,t-2} \cdot ... \cdot DMA_{1,0} \cdot DMA_0$ with $DMA_0 = 1$, and similarly for the other growth factors.

Chart 1: Terms of Trade, Real Exchange Rate, and Trade Balance, Switzerland, 1970-2019 (1970 = 1.0)



it we report the values of the pure terms-of-trade effect (H_t), and the corresponding real-exchange-rate effect (E_t).[26] In the fourth column, one finds the chained values of the complete trading-gain factor (G_t). For comparison purposes, we also report in columns 5 and 6 estimates of an alternative decomposition of the trading gains briefly discussed in the on-line Appendix, namely the terms-of-trade effect $H_{X,t}$ and the related relative price effect $E_{X,t}$. The corresponding yearly geometric means are reported at the bottom of the table.

Looking first at the values of the terms-of-trade effects, one notes that DMA_t and H_t are fairly well correlated, and they closely reflect the evolution of the terms of trade, weighted by the import and export GDP shares. By 2019, the cumulated effect is somewhat larger for index DMA_t, at nearly 6.5 per cent of real GDP, and just short of 6 per cent for H_t. Chart 2 shows the path of these two measures over the sample period. The deviations between the two indices are largest between 2003 and 2015, a period during which the terms of trade were pretty steady, but with the trade imbalance increasing almost continuously. This is when the non-zero price homogeneity of DMA_t comes into play. H_t and $H_{X,t}$, on the other hand, are highly correlated throughout the sample period.

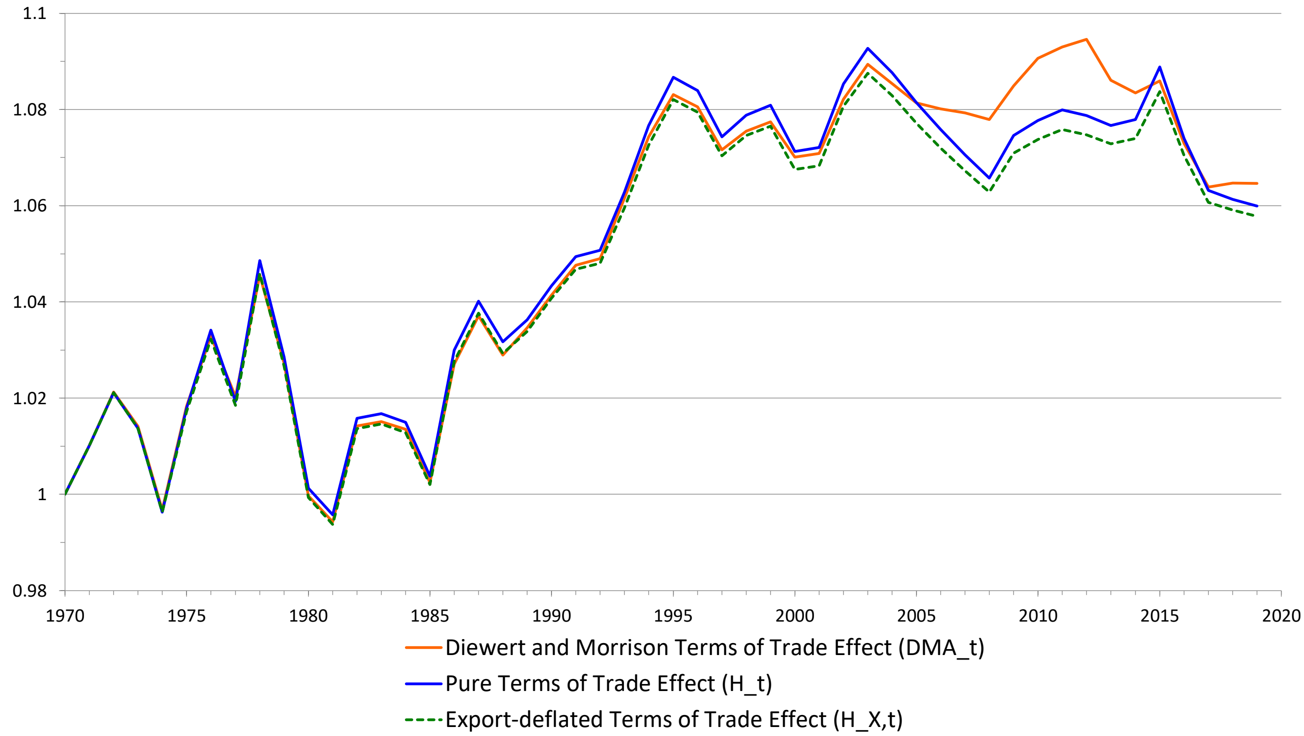
26 For this purpose λ was set to the sample-mean value of s_X, namely 0.5248.

Table 1: Alternative Measures of the Terms-of-Trade, Real-Exchange-Rate, and Trading-Gain Effects, Switzerland, 1970-2019

Year	DMA_t (1)	H_t (2)	E_t (3)	G_t (4)	$H_{X,t}$ (5)	$E_{X,t}$ (6)
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.0101	1.0101	1.0000	1.0102	1.0101	1.0000
1972	1.0213	1.0212	0.9999	1.0210	1.0211	0.9999
1973	1.0142	1.0137	0.9997	1.0134	1.0137	0.9997
1974	0.9968	0.9963	0.9997	0.9960	0.9963	0.9997
1975	1.0183	1.0179	0.9986	1.0164	1.0172	0.9993
1976	1.0329	1.0342	0.9965	1.0306	1.0323	0.9984
1977	1.0203	1.0193	0.9980	1.0173	1.0185	0.9988
1978	1.0456	1.0486	0.9937	1.0420	1.0459	0.9963
1979	1.0272	1.0285	0.9943	1.0226	1.0267	0.9961
1980	0.9997	1.0013	0.9941	0.9954	0.9993	0.9960
1981	0.9943	0.9957	0.9947	0.9905	0.9938	0.9967
1982	1.0142	1.0158	0.9941	1.0099	1.0137	0.9963
1983	1.0151	1.0168	0.9941	1.0108	1.0146	0.9963
1984	1.0135	1.0150	0.9940	1.0089	1.0128	0.9961
1985	1.0024	1.0038	0.9934	0.9971	1.0020	0.9951
1986	1.0271	1.0300	0.9917	1.0214	1.0276	0.9940
1987	1.0371	1.0402	0.9914	1.0313	1.0377	0.9938
1988	1.0290	1.0317	0.9914	1.0228	1.0293	0.9937
1989	1.0346	1.0363	0.9919	1.0279	1.0339	0.9942
1990	1.0414	1.0433	0.9913	1.0342	1.0408	0.9937
1991	1.0476	1.0494	0.9910	1.0400	1.0468	0.9935
1992	1.0490	1.0507	0.9905	1.0407	1.0481	0.9930
1993	1.0615	1.0628	0.9905	1.0527	1.0595	0.9935
1994	1.0743	1.0767	0.9896	1.0654	1.0726	0.9933
1995	1.0831	1.0867	0.9885	1.0743	1.0821	0.9928
1996	1.0806	1.0840	0.9888	1.0718	1.0795	0.9929
1997	1.0716	1.0743	0.9895	1.0630	1.0704	0.9932
1998	1.0755	1.0789	0.9890	1.0670	1.0746	0.9929
1999	1.0775	1.0809	0.9889	1.0689	1.0765	0.9929
2000	1.0701	1.0713	0.9903	1.0609	1.0675	0.9938
2001	1.0709	1.0721	0.9898	1.0611	1.0683	0.9933
2002	1.0821	1.0853	0.9881	1.0725	1.0807	0.9924
2003	1.0894	1.0927	0.9876	1.0792	1.0875	0.9924
2004	1.0854	1.0877	0.9883	1.0749	1.0829	0.9926
2005	1.0814	1.0815	0.9897	1.0703	1.0772	0.9937
2006	1.0801	1.0759	0.9928	1.0681	1.0720	0.9964
2007	1.0793	1.0706	0.9954	1.0657	1.0672	0.9985
2008	1.0779	1.0657	0.9965	1.0620	1.0628	0.9992
2009	1.0849	1.0746	0.9948	1.0690	1.0710	0.9982
2010	1.0906	1.0777	0.9970	1.0745	1.0738	1.0006
2011	1.0930	1.0799	0.9971	1.0768	1.0758	1.0009
2012	1.0946	1.0788	1.0002	1.0790	1.0748	1.0039
2013	1.0861	1.0767	0.9942	1.0705	1.0729	0.9978
2014	1.0834	1.0779	0.9909	1.0681	1.0740	0.9945
2015	1.0860	1.0889	0.9847	1.0722	1.0838	0.9893
2016	1.0729	1.0740	0.9866	1.0596	1.0705	0.9899
2017	1.0639	1.0632	0.9877	1.0501	1.0607	0.9900
2018	1.0647	1.0614	0.9890	1.0497	1.0591	0.9911
2019	1.0647	1.0599	0.9901	1.0494	1.0578	0.9921
Mean (1970-2019)	1.0013	1.0012	0.9998	1.00010	1.0012	0.9998

Note:
DMA_t: Diewert and Morrison terms-of-trade effect (equation 7)
H_t: Terms-of-trade effect holding $e_t = p_{T,t}/p_{N,t}$ constant (equation 14)
E_t: Real-exchange-rate effect (equation 15)
G_t: Trading gains (equation 16)
$H_{X,t}$: Terms-of-trade effect holding $p_{X,t}/p_{N,t}$ constant (equation A1)
$E_{X,t}$: Relative-price effect (equation A2)
Note that: $G_t = H_t \cdot E_t = H_{X,t} \cdot E_{X,t}$ by (16) and (A3).
Values presented in the bottom row are geometric means.

Chart 2: Alternative Measures of the Terms-of-Trade Effects, Switzerland, 1970-2019 *(as factors of real GDP)* (1970 = 1.0)

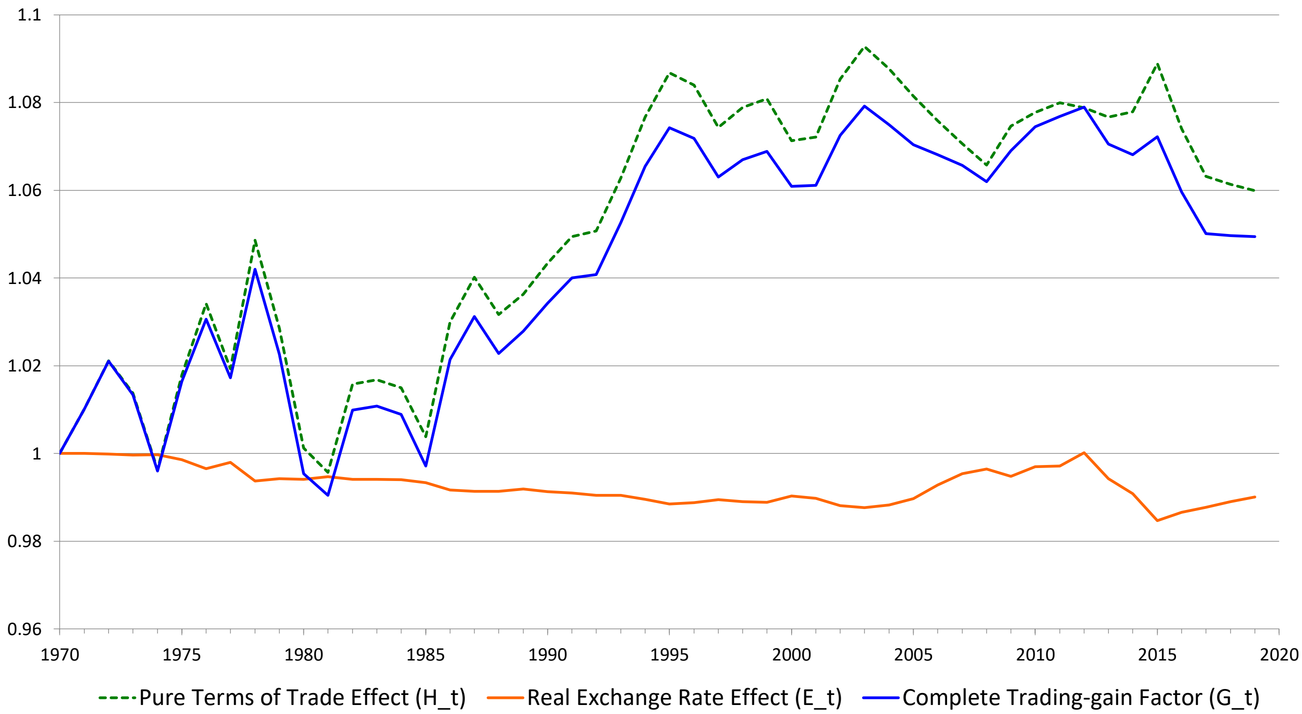


Returning to Table 1, one sees that, by the end of the sample period the cumulated real-exchange-rate effect (E_t) is negative, at about -1.0 per cent. This negative income effect is due to the conjunction of a declining real exchange rate (i.e. a real appreciation) and of a positive trade balance, on average. A similar picture emerges if one considers the relative-price effect $E_{X,t}$, which refers to the price of exports relative to the price of nontraded goods. The total trading gains, finally, show a gain of close to 5 per cent of GDP by 2019. In terms of 2019 prices, this amounts to nearly 36 billion Swiss francs. Admittedly, this is the result of trading gains chained over a 50-year period. Nonetheless, the amount is sizable, particularly if one cumulated the yearly real gains over this period, year after year (using an appropriate real interest rate), and, moreover, considering that for the average country the trading gains must be nil![27]

The trading-gain index, together with our preferred measures of the terms-of-trade and real-exchange-rate components, is depicted in Chart 3. The long-run trends are clearly visible, and so are the shorter-run fluctuations. Looking in more details at the changes through time, the 1985-2005 period stands out. This is when the terms-of-trade effect increased substantially and almost continuously, adding as much as 1.2 percentage points to economic

27 For a sample of 24 OECD member countries covering the period 1970-2012, Switzerland ranked third (behind Australia and Norway) for the relative size of its 2012 trading gains; in terms of the capitalized sum over the entire period, using a 1 per cent real rate of interest, Switzerland came up first with a gain amounting to 168 per cent of its 2012 GDP; see Kohli (2014).

Chart 3: Terms-of-Trade, Real-Exchange-Rate, and Trading-Gain Effects, Switzerland, 1970-2019 *(as factors of real GDP)* (1970 = 1.0)



growth in 2002, and peaking at over 9.3 per cent of GDP a year later. This effect was somewhat dampened by the simultaneous, negative real-exchange-rate effect, but the trading-gains index still reached a high of nearly 8 per cent in 2003, and remained above the 6 per cent mark until 2015.

It is ironic that it is precisely at the beginning at the new millennium, when the trading gains were reaching new highs, that a sense of growth pessimism became prevalent among Swiss economic actors and observers, coming to a climax at a March 2005 conference held in Zurich and organized by the *Avenir Suisse* think tank.[28] The OECD had just published a report widely interpreted as indicating that Ireland had overtaken Switzerland in terms of real income per capita (Wyplosz, 2005), whereas the data referred in fact to real GDP at purchasing-power-parity exchange rates.[29] Once this confusion exposed, it became apparent that the Swiss economic performance was only half as bad as it looked, and that what could be called the Swiss *growth paradox* — which has Switzerland growing less rapidly than most other countries, and yet always remaining among the front runners in terms of real income per capita — could be explained in parts by the official and public fixation on real GDP, as opposed to real GDI and, even

28 I am grateful to Gerhard Schwarz, the then Director of Avenir Suisse, for having invited me to this conference and for his continuous support.

29 Even though the title of the OECD press release indicated that the comparison pertained to GDP figures, the OECD itself referred to income after just two paragraphs.

more importantly, real Gross National Income (GNI).[30]

We report in Table 2, first column, our estimates of TFP (R_t), as given by (5), cumulated over the entire period. The index, set to unity in 1970, reaches a level of 1.564 by the end of the sample period; this implies an annual average contribution to growth of about 0.92 percentage points. In the second column we report our estimates of the contribution of capital deepening, K_t, as given by (23): its contribution over the sample period is nearly as important as that of TFP, with a 2019 estimate of 1.482 (about 0.81 per cent per year on average). Column 3 shows the estimate of $A_{Y,t}$, the average labour productivity defined with respect to real GDP as given by (25), reaching a level of 2.318 by 2019. The next contributing factor is made up by the trading gains, G_t, as discussed above and reported once again in the fourth column of the table for the sake of completeness. Together, R_t, K_t, and G_t explain the growth in the complete, "globalized," measure of average labour productivity ($A_{Z,t}$), shown in the fifth column. It is found to have well more than doubled over five decades, averaging an annual growth rate of 1.83 per cent. The sixth column of the table shows the value of $S_{L,t}$: it increased by about one tenth over the course of the last half-century. The last column, finally, documents the growth in the real marginal product of labour (U_t). It reached a level of 2.672 by 2019 (just over 2 per cent annually), and it thus exceeded the growth of the average labour productivity $A_{Z,t}$ measure by about 0.2 percentage points per year.

Our results are summarized graphically by Chart 4. Starting at the bottom of the graph, we first show the path of TFP (R_t); this line is next augmented by the path of the capital-deepening contributing factor (K_t) to obtain the path of average labour productivity in terms of real GDP ($A_{Y,t}$); next we have added the contribution of the trading gains to obtain the path of average labour productivity in terms of real GDI ($A_{Z,t}$); finally, multiplying by the labour share index ($S_{L,t}$), we get the path of the real wage rate, interpreted as the marginal product of labour. It is quite clear that the two main engines of growth of the Swiss economy are the increases in TFP and capital deepening. As expected, the contribution of the trading gains is much smaller, although not insignificant. Thus, in the Swiss case, trading gains have contributed close to 0.1 per cent annually to the growth in real wages. In any case, good accounting practices require that this component not be overlooked.

Concluding Comments

As shown above, trading gains are important not just for the measurement of real GDI and the determination of aggregate demand, but also for some measures of productivity when defined in a broad context. We have argued that both the measurement of the average and of the marginal productivity of labour should take trading gains into account since al-

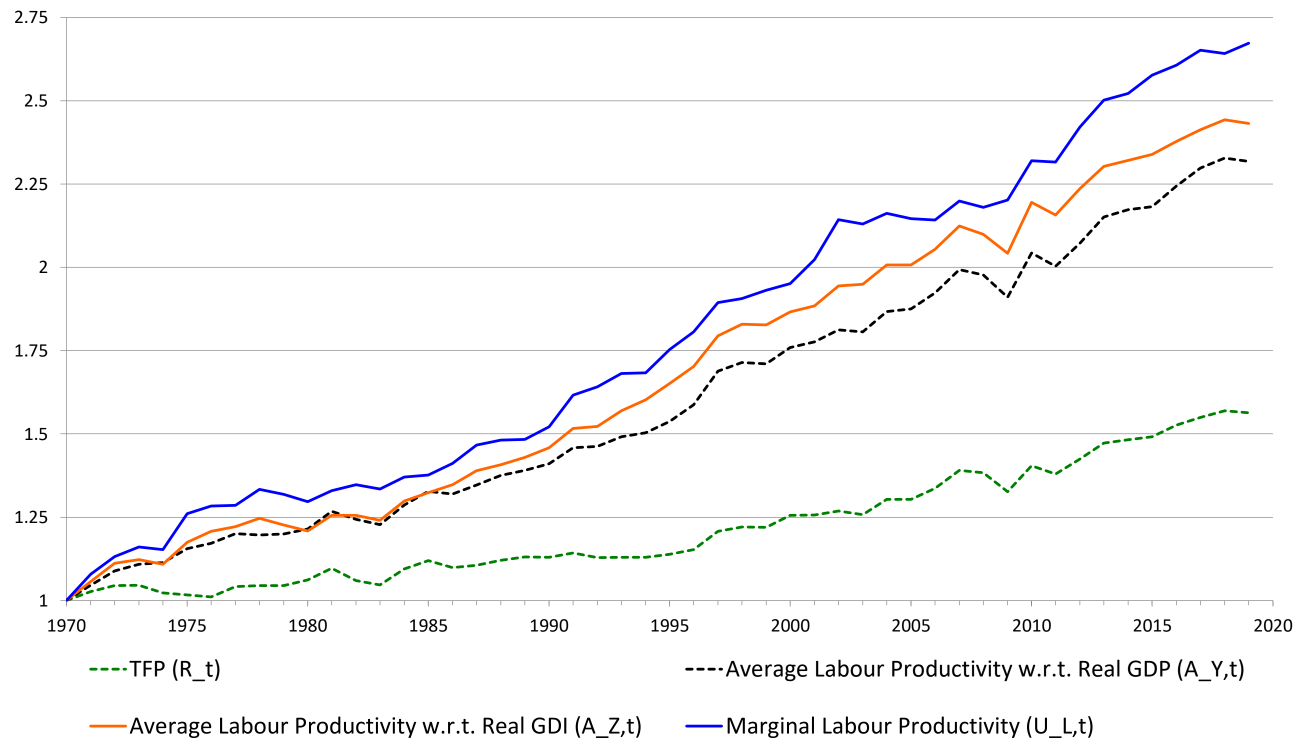
30 See Kohli (2005a); the reactions were virulent, as shown by the following headlines "Krach in der Nationalbank um die Wachstumspolitk" (*Sonntags Zeitung*, March 6, 2005) and "Swiss Pour Scorn on Ireland's Fourth Place in World's Wealthy Elite" (*Irish Times*, March 6, 2005).

Table 2: Alternative Measures of Productivity, Switzerland, 1970-2019

Year	R_t (1)	K_t (2)	$A_{Y,t}$ (3)	G_t (4)	$A_{Z,t}$ (5)	$S_{L,t}$ (6)	$U_{L,t}$ (7)
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.0266	1.0185	1.0456	1.0102	1.0562	1.0208	1.0782
1972	1.0448	1.0416	1.0883	1.0210	1.1112	1.0187	1.1320
1973	1.0453	1.0602	1.1083	1.0134	1.1231	1.0335	1.1607
1974	1.0228	1.0885	1.1133	0.9960	1.1088	1.0394	1.1525
1975	1.0169	1.1361	1.1553	1.0164	1.1743	1.0731	1.2601
1976	1.0107	1.1594	1.1718	1.0306	1.2077	1.0625	1.2831
1977	1.0418	1.1522	1.2004	1.0173	1.2211	1.0525	1.2852
1978	1.0446	1.1457	1.1968	1.0420	1.2471	1.0695	1.3337
1979	1.0448	1.1482	1.1996	1.0226	1.2267	1.0747	1.3184
1980	1.0620	1.1437	1.2146	0.9954	1.2090	1.0727	1.2968
1981	1.0968	1.1555	1.2673	0.9905	1.2552	1.0594	1.3298
1982	1.0600	1.1731	1.2435	1.0099	1.2558	1.0727	1.3470
1983	1.0464	1.1730	1.2274	1.0108	1.2407	1.0755	1.3344
1984	1.0946	1.1758	1.2870	1.0089	1.2985	1.0552	1.3702
1985	1.1194	1.1860	1.3276	0.9971	1.3238	1.0402	1.3770
1986	1.0989	1.2006	1.3193	1.0214	1.3476	1.0472	1.4111
1987	1.1055	1.2184	1.3469	1.0312	1.3890	1.0557	1.4663
1988	1.1210	1.2274	1.3758	1.0228	1.4072	1.0525	1.4811
1989	1.1302	1.2303	1.3905	1.0279	1.4293	1.0376	1.4830
1990	1.1294	1.2488	1.4104	1.0342	1.4587	1.0428	1.5212
1991	1.1426	1.2763	1.4582	1.0400	1.5165	1.0656	1.6159
1992	1.1291	1.2956	1.4629	1.0407	1.5225	1.0779	1.6411
1993	1.1295	1.3203	1.4913	1.0526	1.5698	1.0712	1.6816
1994	1.1300	1.3308	1.5038	1.0654	1.6021	1.0509	1.6837
1995	1.1382	1.3503	1.5370	1.0743	1.6511	1.0619	1.7533
1996	1.1529	1.3773	1.5879	1.0718	1.7018	1.0612	1.8060
1997	1.2075	1.3980	1.6882	1.0630	1.7946	1.0552	1.8937
1998	1.2201	1.4050	1.7143	1.0670	1.8291	1.0422	1.9062
1999	1.2198	1.4017	1.7098	1.0689	1.8275	1.0568	1.9313
2000	1.2556	1.4011	1.7591	1.0609	1.8663	1.0454	1.9510
2001	1.2567	1.4131	1.7758	1.0611	1.8844	1.0734	2.0227
2002	1.2684	1.4291	1.8126	1.0725	1.9439	1.1024	2.1430
2003	1.2579	1.4358	1.8061	1.0792	1.9492	1.0926	2.1297
2004	1.3036	1.4324	1.8672	1.0749	2.0070	1.0770	2.1615
2005	1.3035	1.4387	1.8753	1.0703	2.0073	1.0689	2.1456
2006	1.3368	1.4384	1.9230	1.0681	2.0539	1.0428	2.1417
2007	1.3903	1.4336	1.9932	1.0656	2.1240	1.0354	2.1993
2008	1.3832	1.4291	1.9768	1.0620	2.0993	1.0387	2.1805
2009	1.3264	1.4400	1.9099	1.0690	2.0418	1.0783	2.2016
2010	1.4045	1.4544	2.0427	1.0745	2.1947	1.0571	2.3201
2011	1.3792	1.4526	2.0035	1.0768	2.1574	1.0734	2.3157
2012	1.4249	1.4543	2.0723	1.0790	2.2360	1.0825	2.4205
2013	1.4725	1.4606	2.1508	1.0705	2.3024	1.0867	2.5020
2014	1.4824	1.4656	2.1726	1.0681	2.3205	1.0865	2.5213
2015	1.4912	1.4631	2.1817	1.0722	2.3392	1.1015	2.5767
2016	1.5262	1.4703	2.2439	1.0596	2.3776	1.0963	2.6066
2017	1.5496	1.4829	2.2980	1.0501	2.4132	1.0987	2.6513
2018	1.5695	1.4830	2.3276	1.0497	2.4432	1.0812	2.6417
2019	1.5637	1.4822	2.3177	1.0494	2.4322	1.0987	2.6722
Mean (1970-2019)	1.0092	1.0081	1.0173	1.0010	1.0183	1.0019	1.0203

Note:
R_t: Total Factor Productivity (equation 5)
K_t: Capital intensity (equation 23)
$A_{Y,t}$: Average labour productivity w.r.t. real GDP (equation 25)
G_t: Trading gains (equation 16)
$A_{Z,t}$: Average labour productivity w.r.t. real GDI (equation 21)
$S_{L,t}$: Labour share factor (equation 28)
$U_{L,t}$: Real wage (equation 27)
Note that: $A_{Y,t} = R_t \cdot K_t, A_{Z,t} = R_t \cdot K_t \cdot G_t$, and $U_{L,t} = R_t \cdot K_t \cdot G_t \cdot S_{L,t} = A_{Z,t} \cdot S_{L,t}$ by (16), (24), (25), (26), and (29).
Values presented in the bottom row are geometric means.

Chart 4: Decomposition of the Marginal Productivity of Labour, Switzerland, 1970-2019 (1970 = 1.0)



most all trade takes place during — rather than after — production. Some domestic labour is involved in almost all transactions with the rest of the world, and international trade is an intimate part of production in a globalized world. The distinction between capital deepening, technological progress, human capital enhancement, and trading gains can be blurred. Some advances could be wrongly attributed to one growth factor rather than to another. This calls for an all-encompassing approach where all income-augmenting forces are considered jointly. In fact, when it comes to the marginal productivity of labour, defining the real wage in terms of anything but the purchasing power of domestic income would make little sense.

It is disappointing that the IMF, the OECD, EuroStat, and the United Nations, among others, do not have the resolution to make explicit recommendations concerning the appropriate trade-balance deflator, basically leaving member countries in the dark as to what the best practices are.[31] Thus, it is up to them whether they want to use $p_{M,t}, p_{X,t}, p_{A,t}, p_{N,t}$, or yet another price index, as a deflator. Moreover, unless trade happens to be balanced, all the so-called measures of the trading gains using a deflator other than $p_{N,t}$ are incomplete since they exclude the relative-price effect resulting from a change in the price of the chosen trade-account deflator relative to the price of domestic final goods. This is why additional components such as

31 Admittedly, the IMF has been advocating the use of the price of gross domestic final expenditures to compute real GDI in some of its own policy work; see Reinsdorf (2020, paragraphs 30, 87).

the real exchange rate effect, $E_{t,t-1}$, are needed. Thus, these official measures are misnamed: they should be viewed at best as measures of the terms-of-trade effects, rather than of the full trading gains. Consequently, the corresponding real GDI estimates must be considered as flawed.

It would appear that most statistical agencies have it backwards. They select a deflator, more or less at random, receiving no strict guidance from the SNA. They then very carefully calculate the (incomplete) trading gain, add it to their estimate of real GDP, and declare it to be real GDI. The implicit GDI deflator is then almost meaningless since it will generally be a function of the prices of imports and/or exports, incorrectly suggesting that a change in the prices of traded goods would change real domestic income for a *given nominal domestic income and a given domestic price level*. Real GDI then becomes some kind of curiosity in the system of national accounts, with no obvious link to the other aggregates. Instead, these agencies and the authors of the SNA should begin by asking themselves what real GDI is supposed to measure. In our view, it should be the real purchasing power that is available domestically, at price $p_{N,t}$.[32] Once that nominal GDP has been deflated in that way to yield real GDI, it is straightforward to compute the trading gain as the ratio of the GDP deflator to the domestic final expenditure price index as shown by (19). The trading gain can then be decomposed into terms-of-trade and real-exchange-rate effects as shown above. This is so simple that it is hard to understand why real GDI and the trading-gain concepts are not standard elements of the macroeconomic toolbox. One can only hope that in its next revision, due in 2025, the SNA will provide definite guidance as to what the best practice is.[33]

Today, Statistics Canada and the Bank of Canada stand out as being the only institutions, to the best of our knowledge, that not only publish real GDI using the price of domestic final expenditures as a deflator, together with the trading-gain estimate, but also the corresponding terms-of-trade and the real-exchange-rate components. As the French saying goes, “nul n’est prophète en son pays!” The Swiss National Bank, that was possibly the very first official institution to publish real GDI statistics using $p_{N,t}$ as the deflator starting in July 2007, inexplicably stopped publishing them in October 2014.

32 See Kohli (2004a:97); Reinsdorf (2010, 2020) expresses the same opinion. Oulton (2004), Diewert and Lawrence (2006), and Sefton and Weale (2006) favour the use of the price of private consumption goods as the deflator. However, we see little merit in excluding government expenditures (public consumption) and investment (deferred consumption) since these make up about half of domestic final expenditures, if not more, in many countries. As noted by Reinsdorf, in the absence of evidence to the contrary, it is reasonable to assume that the marginal income arising from trading gains is spent in the same way as average income.

33 There are of course numerous other — and undoubtedly more important — aspects of the System of National Accounts (SNA) that are subject to regular criticism and recommendations, particularly when it comes to the treatment of non-market activities, externalities, the use of natural resources, and many non-economic issues. Our point, though, is that a significant improvement could be made here at basically zero cost.

References

Boulat, R. (2006) "La Productivité et sa Mesure en France (1944-1955)," *Histoire & Mesure*, Vol. XXI, No.1, pp. 79-110.

Burge, R.W. and R.C. Geary (1957) "Balancing of a System of National Accounts in Real Terms," Chapter 2 in United Nations Economic and Social Council, *A System of Price and Quantity Indexes for National Accounts*, E/CN.3/L.46.

Christensen, L.R., D.W. Jorgenson, and L.J. Lau (1973) "Transcendental Logarithmic Production Frontiers," *Review of Economics and Statistics*, Vol. 55, pp. 28-45.

Corden, W.M. (1960) "The Geometric Representation of Policies to Attain Internal and External Balances," *Review of Economic Studies*, Vol. 28, pp. 1-22.

Corden, W.M. (1992) "Dependent Economy Model of the Balance of Payments," *New Palgrave Dictionary of Money and Finance* (London: Macmillan).

Denison, E.F. (1981) "International Transactions in Measures of the Nation's Production," *Survey of Current Business*, Vol. 61, pp. 17-28.

Diewert, W.E. (1974) "Applications of Duality Theory," in Michael D. Intriligator and David A. Kendrick (eds.) *Frontiers of Quantitative Economics*, Vol. 2 (Amsterdam: North-Holland).

Diewert, W.E. (1976) "Exact and Superlative Index Numbers," *Journal of Econometrics*, Vol. 4, pp. 115-145.

Diewert, W. E. and D. Lawrence (2006) *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, report by Meyrick and Associates to the Productivity Commission, Canberra.

Diewert, W. E. and C. J. Morrison (1986) "Adjusting Output and Productivity Indices for Changes in the Terms of Trade," *Economic Journal*, Vol. 96, pp. 659-679.

European Commission (2013) *European System of Accounts, ESA 2010* (Luxembourg: Publications Office of the European Union). https://ec.europa.eu/eurostat/documents/3859598/5925693/KS-02-13-269-EN.PDF/44cd9d01-bc64-40e5-bd40-d17df0c69334

Fox, K.J. and U. Kohli (1998) "GDP Growth, Terms-of-Trade Effects, and Total Factor Productivity," *Journal of International Trade and Economic Development*, Vol. 7, pp. 87-110.

Fox, K.J., U. Kohli, and R.S. Warren Jr. (2002) "Accounting for Growth and Output Gaps: Evidence from New Zealand," *Economic Record*, Vol. 78, pp. 312-326.

Geary, R.C. (1961) "Problems in the Deflation of National Accounts: Introduction," *Review of Income and Wealth*, Vol. 9, pp. 3-8.

Hartwick, J.M. (2020) *Advanced Introduction to National Accounting* (Cheltenham, United Kingdom: Edward Elgar).

International Monetary Fund (2009a) *System of National Accounts 2008* (New York: International Monetary Fund).

International Monetary Fund (2009b) *Export and Import Price Index Manual: Theory and Practice* (Washington, D.C.: International Monetary Fund).

Kehoe, T.J. and K.J. Ruhl (2008) "Are Shocks to the Terms of Trade Shocks to Productivity?," *Review of Economic Dynamics*, Vol. 11, pp. 804-819.

Kohli, U. (1978) "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports," *Canadian Journal of Economics*, Vol. 11, pp. 167-182.

Kohli, U. (1990) "Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates," *Journal of Economic and Social Measurement*, Vol. 16, pp. 125-136.

Kohli, U. (1991) *Technology, Duality, and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports* (Ann Arbor, MI: University of Michigan Press).

Kohli, U. (1993) "A Symmetric Normalized Quadratic GNP Function and the U.S. Demand for Imports and Supply of Exports," *International Economic Review*, Vol. 34, pp. 243-255.

Kohli, U. (2002) "Terms-of-Trade Changes and Real GDP: Evidence from Switzerland," *Quarterly Bulletin of the Swiss National Bank*, pp. 55-63.

Kohli, U. (2003) "Terms of Trade, Real GDP, and Real Value Added: A New Look at New Zealand's Growth Performance," *New Zealand Economic Papers*, Vol. 37, pp. 41-66.

Kohli, U. (2004a) "Real GDP, Real Domestic Income, and Terms-of-Trade Changes," *Journal of International Economics*, Vol. 62, pp. 83-106.

Kohli, U. (2004b) "An Implicit Törnqvist Index of Real GDP," *Journal of Productivity Analysis*, Vol. 21, pp. 337-353.

Kohli, U. (2005a) "Switzerland's Growth Deficit: A Real Problem – But Only Half as Bad as it Looks", in Lukas Steinmann und Hans Rentsch (eds.) *Diagnose: Wachstumsschwäche* (Zurich: Verlag Neue Zürcher Zeitung).

Kohli, U. (2005b) "Labour Productvity vs. Total Factor Productivity," *Irving Fisher Committee Bulletin*, Vol. 20, pp. 106-116.

Kohli, U. (2006a) "Terms of Trade, Real Exchange Rates, and Trading Gains," SSHRC Conference on Price and Productivity Measurement, Vancouver, B.C., Canada, July 2004.

Kohli, U. (2006b) "Real GDP, Real GDI, and Trading Gains: Canada, 1981-2005," *International Productivity Monitor*, Vol. 13, pp. 46-56. http://www.csls.ca/ipm/13/IPM-13-kohli-e.pdf

Kohli, U. (2007) "Terms-of-Trade Changes, Real GDP, and Real Value Added in the Open Economy: Reassessing Hong Kong's Growth Performance," *Asia-Pacific Journal of Accounting and Economics*, Vol. 14, pp. 87-109.

Kohli, U. (2010) "Labour Productivity: Average vs. Marginal," in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.) *Price and Productivity Measurement*, Vol. 6 (Victoria, B.C.: Trafford Press).

Kohli, U. (2014) "Who Gains Most from Trade?," paper presented at the EMG Workshop, University of New South Wales, Sydney, December 2014.

Kohli, U. and J.-M. Natal (2014) "The Real Exchange Rate and the Structure of Aggregate Production," *Journal of Productivity Analysis*, Vol. 42, pp. 1-13.

Landefeld, J.S. and R.P. Parker (1997) "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth," *Survey of Current Business*, July.

Macdonald, R. (2010) "Real Gross Domestic Income, Relative Prices and Economic Performance Across the OECD," *Review of Income and Wealth*, Vol. 56, pp. 498-518.

Macdonald, R. (2020) "Measurement of Real Income in the System of National Accounts: An Application to North American Economies," Economic Analysis Research Paper Series (Ottawa, Ontario: Statistics Canada).

Macdonald, R. and L. Rispoli (2016) "How Big was the Effect of Falling Commodity Prices on Canadian Real Incomes Between Mid-2014 and Mid-2016?," *Economic Insights* (Ottawa, Ontario: Statistics Canada).

Neary, J.P. (1997) "R.C. Geary's Contributions to Economic Theory," Working Paper 97/04, University College Dublin, January.

Oulton, N. (2004) "Productivity versus Welfare; or GDP versus Weitzman's NDP," *Review of Income and Wealth*, Vol. 50, pp. 329-355.

Reinsdorf, M.B. (2010) "Terms of Trade Effects: Theory and Measurement," *Review of Income and Wealth*, Vol. 56, S177-S205.

Reinsdorf, M.B. (2020) "Measuring Economic Welfare: What and How?," IMF Policy Paper, March.

Salter, W.E.G. (1959) "Internal and External Balance: The Role of Price and Expenditure Effects," *Economic Record*, Vol. 35, pp. 226-238.

Sanyal, K.K. and R.W. Jones (1982) "The Theory of Trade in Middle Products," *American Economic Review*, Vol. 72, pp. 16-31.

Sefton, J.A. and M.R. Weale (2006) "The Concept of Income in General Equilibrium," *Review of Economic Studies*, Vol. 73, pp. 219-249.

Statistics Canada (2016) *User Guide: Canadian System of Macroeconomic Accounts* (Ottawa, Ontario: Statistics Canada).

Stiglitz, J.E., A. Sen, and J.-P. Fitoussi (2009) *Report* (Paris: Commission on the Measurement of Economic Performance and Social Progress).

Swiss National Bank (2007) "Gross Domestic Product, Trading Gains and Gross National Income," *SNB Monthly Statistical Bulletin*. July.

Woodland, A.D. (1982) *International Trade and Resource Allocation* (Amsterdam: North-Holland).

Wyplosz, C. (2005) "C'est Maintenant Confirmé: le Revenu par Tête (Corrigé pour le Coût de la Vie) Irlandais a Dépassé le Niveau Suisse en 2002," *Le Temps*, January 31. https://www.letemps.ch/economie/recettes-miracle-irlandais-un-exemple-suisse.