On the Relationship between Gross Output-based TFP Growth and Value Added-based TFP Growth: An Illustration Using Data from Australian Industries

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ABSTRACT

Diewert (2015) develops simple expressions for the exact relationship between value added TFP growth and gross output TFP growth. These expressions suggest that the magnification factor relating the two TFP growth measures is approximately equal to the share of primary inputs in total costs. We apply these simple approximations to data on Australian industries, finding that they tend to provide very close approximations over short time periods, but are less reliable over longer time horizons. We find that magnification factors vary significantly across industries so that the results of comparative studies can be quite sensitive to the choice of output measure chosen to construct TFP. In particular, industries in which intermediate inputs account for a large share of total inputs exhibit much smaller TFP growth (in absolute value) compared to industries where intermediate inputs are relatively unimportant when gross output is used to construct TFP measures rather than value added.

IT HAS LONG BEEN KNOWN THAT the measurement of total factor productivity (TFP)[2] can be sensitive to the choice of input and output measures. Researchers have previously derived mathematical relationships between TFP calculated using value added to measure output and TFP calculated using gross output (Schreyer, 2001; Balk, 2009). In the preceding article, Diewert (2015) presents new expressions relating these two approaches to measuring TFP growth under Laspeyres, Paasche, and Fisher index number frameworks which are simpler than those previously available in the literature. The gross output TFP growth rate is equal to the value added TFP growth rate multiplied by a simple magnification factor. The magnification factor is the share of primary inputs (capital and labour) in total input use[3]

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2 The term "total factor productivity" is more commonly used by academics while statistical agencies tend to refer to "multifactor productivity" (MFP). The two terms refer to the same concept, the extent to which output growth exceeds input growth. To avoid confusion, we will always use the term TFP in this article except when providing the name of our data source.

multiplied by the ratio of the growth factor[4] of primary inputs to the growth factor of all inputs.[5]

This article addresses two questions arising from the mathematical results of Diewert (2015). Diewert shows that the share of primary inputs in total input use will yield a close estimate of the magnification factor under the assumption that the ratio of growth factors of primary inputs and total inputs will be close to one (i.e. no change in the share of intermediate inputs), and it is suggested that a simple average of the Paasche and Laspeyres magnification factors will be approximately equal to the Fisher magnification factor. This raises our first question, how accurate are these approximations in practice? Our second question is how sensitive are comparisons of TFP growth across industries to the choice of output measure? While the theoretical effect is obvious, we examine whether magnification factors vary enough across industries in practice that this should be a concern in practice.

We explore these empirical questions by constructing magnification factors using industry level productivity data from Australia between 1994 and 2013 and then comparing these to each other and to approximations constructed using the formulae developed by Diewert (2015).

Methodology and Data

Most of the calculations in this article use formulae from Diewert (2015) which we shall briefly describe. For simplicity, we adopt Diewert's notation.[6] For ease of interpretation, the magnification factors which we calculate are the reciprocal of those presented by Diewert. This is so that we can discuss the share of primary inputs in total input costs rather than its inverse.

Diewert (2015, equation 13) derives an exact relationship between gross output TFP growth and value added TFP growth between two years, year 0 and year 1, in a Laspeyres index number framework:

$$\frac{1}{\gamma} = \frac{\pi_G}{\pi_{VA}} = \frac{(1-s_M^0)\Gamma_X}{s_M^0\Gamma_M + (1-s_M^0)\Gamma_X}$$

where $1/\gamma$[7] is the magnification factor, π_G is the growth rate of TFP based on gross output, π_{VA} is the growth rate of TFP based on value added, Γ_X[8] is the growth factor of primary inputs, Γ_M is the growth factor of intermediate inputs, s_M^0 is the nominal share of intermediate inputs in total input use, and $(1-s_M^0)$ is the share of primary inputs in total input use. A superscript 0 indicates that the value from the initial period, period 0, is being used.

This expression has a very simple interpretation: gross output-based TFP growth is equal to value added-based TFP growth multiplied by the "contribution" of primary inputs, $(1-s_M^0)\Gamma_X$, to the growth factor for all inputs, $s_M^0\Gamma_M + (1-s_M^0)\Gamma_X$. If intermediate inputs make no contribution to total input growth (i.e. the share of intermediate inputs is zero), then the TFP growth rates based on

3 In nominal terms, the value of all inputs is equal to the value of gross output. The share of primary inputs in total input use is equal to one minus the share of intermediate inputs in total input use.

4 A growth factor is equal to 1 plus the growth rate. For example, let γ_t represent income at time t. A growth rate of income of 4 per cent between years t and t+1 corresponds to a growth factor of $\gamma_{t+1}/\gamma_t = 1.04$.

5 In a Paasche index framework, the magnification factor is the share of primary inputs multiplied by the ratio of the growth factor of gross output growth to the growth factor of real value added growth. The Fisher magnification factor can be approximated by taking an appropriately weighted average of the Paasche and Laspeyres magnification factors.

6 For consistency with the terminology used in Diewert (2015), we will refer to the term as a magnification factor even though it scales down the value added-based TFP estimates in this article.

7 γ is the Greek letter "gamma" (lowercase).

8 Γ is the Greek letter "gamma" (uppercase).

value added and gross output will be identical. If primary inputs make a very small contribution to total input growth, then the growth rate of TFP based on value added will be considerably larger than that based on gross output. Note that the absolute value of the TFP growth rate based on gross output will always be smaller than that based on value added provided that the growth factors of primary and intermediate inputs and the share of intermediate inputs in total inputs are all positive, which will almost always be the case.[9]

Notice that the magnification factor can be broken down into two multiplicative terms, the share of primary inputs in total input costs, $(1-s_M^0)$, and the ratio of the growth factor of primary inputs to the growth factor of all inputs, $\frac{\Gamma_X}{s_M^0\Gamma_M + (1-s_M^0)\Gamma_X}$. If this ratio of growth factors is not equal to one, then this implies that the real cost shares of primary inputs and intermediate inputs are changing through time.[10] Diewert (2015) points out that if the growth factors of primary inputs Γ_X and total inputs $s_M^0\Gamma_M + (1-s_M^0)\Gamma_X$ are very similar, then their ratio will be close to 1. This will happen if both growth rates are fairly small in absolute valie, for example. Consequently, a simpler approximation will often be sufficient. The magnification factor is approximately equal to the share of primary inputs in total input use at time 0:

$$\frac{\pi_G}{\pi_{VA}} \approx (1-s_M^0)$$

Notice that this approximation implies that the growth rate of TFP based on gross output will normally be less in absolute value than that based on value added. Intuitively, the ratio between the real value of gross output and the real value of all inputs will be closer to 1 than the ratio between value added and primary inputs because the former is identical to the latter, but with some positive real value of intermediate inputs added to both the numerator and denominator.[11]

Suppose, for example, that primary input grows by 1 per cent, value added grows by 2 per cent, and gross output grows by 2 per cent. In this case, value added-based TFP would grow by 1 per cent. Notice that gross output-based TFP must grow by less than 1 per cent. Why? Because gross output is equal to value added plus intermediates, and both gross output and value added grow by 2 per cent, it must be the case that intermediates grow by 2 per cent as well. If intermediates grow by 2 per cent and primary inputs by 1 per cent, then the growth rate of all inputs must lie somewhere between 1 and 2 per cent, depending upon the relative importance of primaries and intermediates. This means that TFP growth under gross output will be no higher than TFP growth under value added, lying somewhere between 0 and 1 per cent. If primary inputs account for most of the input costs, then TFP growth will be closer to 1 per cent. If intermediate inputs are very important, TFP growth will be closer to 0 per cent. Generally speaking, any change in the ratio between value added and primary inputs is weakened by adding the same amount of intermediates to both the numerator and denominator in a gross output framework, bringing the TFP growth rate closer to 0 per cent.

We apply approximations of the Laspeyres, Paasche, and Fisher magnification factors to Australian data at the industry level taken from the Australian Bureau of Statistics (ABS) Esti-

9 For this not to be true, there would either need to be non-positive quantities of inputs or non-positive prices.

10 Since the share of primary inputs in total input costs is equal to the ratio of primary input costs to total input costs at time 0 and the second term is the ratio of the growth factors of primary inputs and total inputs, the magnification factor can also be interpreted as simply the real cost share of primary inputs in total input costs at time 1 (i.e. one minus the real share of intermediate inputs in total input costs).

11 Of course, both ratios will be exactly equal to one in the base year, as the nominal value of gross output is equal to the nominal value of total inputs in this year.

mates of Industry Multifactor Productivity.[12] Only the Laspeyres results are presented in the text, but the results under Paasche and Fisher frameworks are included in the online appendix.[13]

In practice, the ABS uses chained Laspeyres quantity indexes for gross output, intermediate inputs, and real value added and chained Tornqvist-Theil indexes as their index number formulae for primary inputs. This means that our formulae will produce results which are inconsistent with the official TFP estimates produced by the ABS because the ABS used a different index number approach to construct their TFP estimates. To avoid this problem, we treat the data from the ABS as "true" estimates of economic activity and use these to perform our own calculations of TFP growth under Laspeyres, Paasche, and Fisher frameworks. In particular, we take the ABS cost shares of intermediate inputs and growth factors of primary inputs, intermediate inputs, and gross output as given and aggregate these data to construct growth factors for value added and total input. Note that this means that the magnification factors we construct under the Laspeyres and Paasche frameworks will be exactly the same as the ratio of gross output-based TFP and value added-based TFP by construction. The advantage of this approach is that we can examine the relative contributions of the two components of the Laspeyres and Paasche magnification factors and the reliability of the Fisher approximations and be confident that any errors are related to the accuracy of the approximations rather than inconsistency between what we are approximating and the data we are using.

To assess the accuracy of the approximations, we calculate the deviation of an approximation from the actual magnification factor as a percentage of the actual magnification factor.

We will compare the magnification factors using the TFP estimates of the ABS to those which we generate given the underlying ABS data using Paasche, Laspeyres, and Fisher aggregation methods when examining the robustness of these magnification factors to the chosen index number framework.

Our analysis will be split into two time periods, one short and one long, in order to explore the accuracy of the various approximations under different time horizons. Intuitively, one would expect that the assumptions underlying the approximations are more likely to hold over shorter time periods. The first time period considers the annual change in 2012.[14] The second time period is 18 years long, from 1994 to 2012. This is the longest time period available from the data source we are using.

Results

A One Year Period, 2011-12

First, consider the shorter time period from 2011 to 2012. The underlying data on the growth rates of gross output, primary inputs, intermediate inputs, and total inputs is presented in Table 1.

Table 2 presents the growth rates of TFP based on gross output and value added and the magnifi-

12 Specifically, we use an index of gross output from Table 16 (to calculate Γ_Y), and index of combined labour and capital inputs from Table 11 (to calculate Γ_X), and index of intermediate inputs from Table 18 (to calculated Γ_M), the cost share of intermediate inputs from Table 19 (to calculate s_M^0 and s_M^1), gross output-based TFP from Table 15, and value added-based TFP from Table 1.

13 The online appendix is available at www.csls.ca/ipm/29/calverappendix.pdf. The methodology for the Paasche and Fisher calculations is reviewed in a short appendix at the end of this article.

14 The ABS reports industry TFP data over fiscal years beginning in April and ending in March, so it is technically 2011/12 – 2012/13. For brevity, we will refer to time periods by the first year only in the text (ie, 1994/95 will be referred to as simply 1994), as most of the time period falls within that calendar year.

Table 1
Growth Rates of Inputs and Output, 2011-2012, and Cost Share of Primary Inputs, 2011 and 2012, Australian Industries

	Growth Rates (%)					Nominal Cost Share of Primary Inputs in Total Input Costs	
	Gross Output	Value Added (Laspeyres Aggregation)	Intermediate Inputs	Primary Inputs	All Inputs (Laspeyres Aggregation)	2011	2012
Sector	(1)	(2)	(3)	(4)	(5)	(6)	(7)
A Agriculture, Forestry and Fishing	-0.60	-0.62	-0.58	-0.81	-0.67	0.41	0.41
B Mining	5.86	8.61	2.23	14.42	9.18	0.57	0.53
C Manufacturing	-2.02	-2.56	-1.81	-1.94	-1.84	0.28	0.28
D Electricity, Gas, Water and Waste Services	1.19	0.66	1.67	-0.20	0.79	0.47	0.48
E Construction	3.02	2.14	3.41	0.43	2.49	0.31	0.31
F Wholesale Trade	0.07	2.39	-1.99	2.68	0.21	0.47	0.48
G Retail Trade	1.88	3.87	-0.85	-0.24	-0.50	0.58	0.59
H Accommodation and Food Services	-0.77	-0.45	-1.07	0.54	-0.29	0.48	0.49
I Transport, Postal and Warehousing	2.69	3.27	2.22	2.15	2.19	0.45	0.44
J Information, Media and Telecommunications	1.72	-0.44	3.49	3.46	3.47	0.45	0.46
K Financial and Insurance Services	4.18	3.70	4.99	-0.73	1.38	0.63	0.62
L Rental, Hiring and Real Estate Services	2.72	4.51	1.00	0.25	0.63	0.49	0.49
M Professional, Scientific and Technical Services	2.85	3.90	1.75	2.35	2.06	0.51	0.52
N Administrative and Support Services	0.44	1.20	-0.49	-2.44	-1.56	0.55	0.54
R Arts and Recreation Services	0.05	0.60	-0.30	2.01	0.60	0.39	0.38
S Other Services	-3.45	-3.85	-2.99	-0.18	-1.50	0.53	0.52

Note: Column (2) =100*[[1+(1)/100]/(6) – [1+(6)]*[1+(3)/100]/(6)-1] and Column (5) = 100*[[1+(3)/100]*[1-(6)]+ [1+(4)/100]*(6) – 1].

Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity.

cation ratio between them under a Laspeyres framework. Notice that the magnification factors are considerably less than one for all sixteen industries under consideration. This indicates that TFP growth is much smaller in absolute value if measured using gross output. For example, the magnification factor of 0.409 in agriculture, forestry, and fishing means that TFP growth is about 2.44 times greater (the reciprocal of 0.409) when measured using value added than when measured using gross output. We should emphasize that TFP growth is smaller in absolute value when measured using gross output, but the growth rate for gross output-based TFP can still be better if TFP growth is negative. For example, accommodation and food services has a TFP growth rate of -0.48 per cent based on gross output, but -0.99 per cent based on value added. Thus, using value added does not necessarily mean that the measured TFP growth will be better, but only that it will be larger in terms of absolute value (i.e. further from 0).

Table 2

Magnification Factors, Gross Output-Based TFP Relative to Value Added-Based TFP in Australian Industries, Laspeyres Index Number Framework, 2011-2012

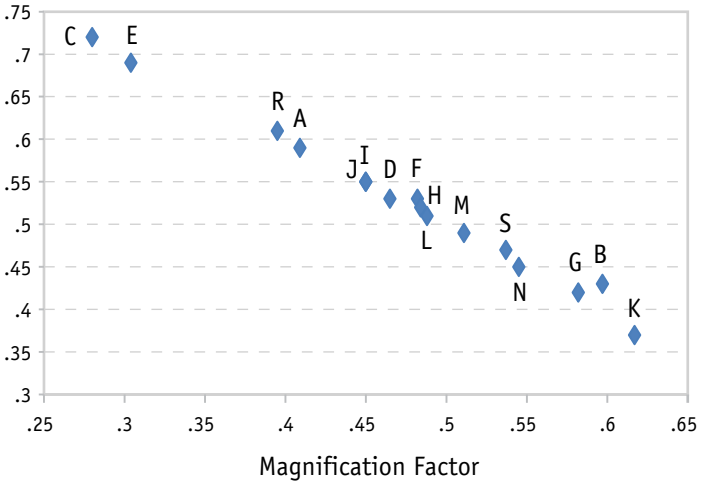
	TFP Growth Rates		Magnification Factor	Components of Magnification Factor		
Industry	Gross Output-Based (%) (1)	Value Added-Based (%) (2)	Ratio of Gross Output-based to Value Added-based TFP Growth Rates (3)=(1)/(2)=(4)*(5)	Cost Share of Primary Inputs in Total Input Costs 2011 (4)	Ratio of Growth Factor of Primary Inputs to Growth Factor of All Inputs (5)	Error of Primary Cost Share Approximation (%) (6) = 100*((4)/(3)-1)
A Agriculture, Forestry and Fishing	0.08	0.19	0.409	0.41	1.00	0.14
B Mining	-3.03	-5.08	0.597	0.57	1.05	-4.58
C Manufacturing	-0.18	-0.63	0.280	0.28	1.00	0.10
D Electricity, Gas, Water and Waste Services	0.40	0.86	0.465	0.47	0.99	0.99
E Construction	0.52	1.70	0.304	0.31	0.98	2.05
F Wholesale Trade	-0.13	-0.28	0.482	0.47	1.02	-2.41
G Retail Trade	2.39	4.12	0.582	0.58	1.00	-0.26
H Accommodation and Food Services	-0.48	-0.99	0.484	0.48	1.01	-0.83
I Transport, Postal and Warehousing	0.49	1.10	0.450	0.45	1.00	0.04
J Information, Media and Telecommunications	-1.70	-3.77	0.450	0.45	1.00	0.02
K Financial and Insurance Services	2.76	4.47	0.617	0.63	0.98	2.13
L Rental, Hiring and Real Estate Services	2.08	4.25	0.488	0.49	1.00	0.38
M Professional, Scientific and Technical Services	0.77	1.51	0.511	0.51	1.00	-0.29
N Administrative and Support Services	2.03	3.73	0.545	0.55	0.99	0.90
R Arts and Recreation Services	-0.55	-1.39	0.395	0.95	1.01	-1.38
S Other Services	-1.98	-3.68	0.537	0.53	1.01	-1.32

Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity

The two multiplicative terms which determine the magnification factor are presented in Table 2 (columns 5 and 6) to highlight the relative importance of the cost share of primary inputs (the initial importance of intermediate inputs) and the primary input growth factor relative to the growth factor for all inputs (the change in the importance of primary inputs in real terms).

The relationship between the share of primary inputs in total input costs and the magnification factor (the first factor) is intuitive. Suppose that primary inputs account for about half of the value of total inputs and that there was positive value added TFP growth in the industry (this was the case in rental, hiring, and real estate services over the 2011-2012 period). This growth in value added-based TFP which is linked to increased productivity of primary inputs will be "watered down" when considering gross output because primary inputs only account for a fraction of total input costs in gross output. For example, if primary inputs only account for half the inputs in gross output, then gross output-based TFP

Chart 1
Nominal Share of Intermediate Inputs in Total Input Costs, 2011, and Magnification Factor, 2012, Australian Industries
(Share of intermediates, per cent)



Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity. Magnification factors produced using a Laspeyres framework. Labels correspond to the industry codes listed in Table 1.

growth should only be about half that of value added-based TFP growth (a magnification factor of about 0.50), which is approximately what we observe in rental, hiring and real estate services. Similarly, if primary inputs accounted for one-third of total input costs, then gross output would only be expected to have TFP growth one third as great as that observed using value added (a magnification factor of about 0.33). The general rule is that the magnification factor is roughly equal to the share of primary inputs in total input costs in the base year provided that primary and intermediate outputs are growing at similar rates. As intermediate inputs become more important, the magnification factor falls, indicating a greater difference between gross output-based and value added-based TFP estimates.

As expected, the ratios of the growth factor of primary inputs to the growth factor of all inputs tends to be close to one so that the share of real intermediate inputs in total input costs remains fairly stable through time. Consequently, the cost share of primary inputs attends to be a good approximation of the magnification factor, with an error of less than one per cent in magnitude in most industries.[15]

The notable exceptions are mining and wholesale trade, where the respective errors are about 4.6 and 2.4 per cent. The relatively poor performance of the approximation for mining is due to growth in primary inputs which was quite a bit faster than growth in all inputs in the industry. This second factor is also somewhat intuitive. Notice that it is related to the first factor, the share of primary inputs in total input costs. If primary inputs grow faster than all inputs, the real share of intermediate inputs in total input costs falls through time, reducing the absolute value of gross output-based TFP growth relative to that of value added-based TFP growth.

The cost shares of primaries, and hence the magnification factors, vary considerably across industries. Financial and insurance services had a very high share of primary inputs in total input costs of 0.63 in 2011 (Table 1). Consequently, TFP growth is only about 63 per cent as fast in this sector if measured using gross output rather than value added. Manufacturing and construction had the lowest shares of primary inputs in total input costs of 0.28 and 0.31 respectively, which results in TFP growth being about 3.6 and 3.3 times greater when measured by value added.

In most cases, the magnification factors range between 0.4 and 0.6, implying that the value added-based TFP estimates are usually between 1.7 and 2.5 times higher than gross output-based TFP estimates. It is important to be aware of this if comparing results generated using the two different approaches. Such results are compara-

15 This simple rule of thumb performs similarly well under a Paasche framework when short periods of time are considered (see Appendix Table 1 in the online appendix).

Table 3
Growth Rates of Inputs and Output, 1994-2012, and Cost Share of Primary Inputs, 2011 and 2012, Australian Industries

	Growth Rates (%)				Cost Share of Primary Inputs in Total Input Costs	
Sector	Gross Output	Intermediate Inputs	Primary Inputs	All Inputs (Laspeyres Aggregation)	1994	2012
	(1)	(3)	(4)	(5)	(6)	(7)
A Agriculture, Forestry and Fishing	46.4	22.7	-0.8	13.1	0.36	0.41
B Mining	82.7	56.7	244.4	163.7	0.56	0.53
C Manufacturing	28.6	32.8	11.3	26.8	0.32	0.28
D Electricity, Gas, Water and Waste Services	23.2	18.6	92.3	53.2	0.58	0.48
E Construction	139.8	133.9	82.1	117.8	0.31	0.31
F Wholesale Trade	111.6	138.3	31.8	88.3	0.53	0.48
G Retail Trade	53.9	16.0	43.4	31.9	0.45	0.59
H Accommodation and Food Services	62.1	63.0	34.9	49.5	0.41	0.49
I Transport, Postal and Warehousing	73.8	58.4	58.6	58.5	0.40	0.44
J Information, Media and Telecommunications	123.8	137.9	106.5	123.8	0.42	0.46
K Financial and Insurance Services	138.8	131.6	68.1	91.6	0.62	0.62
L Rental, Hiring and Real Estate Services	52.7	47.7	179.8	112.4	0.42	0.49
M Professional, Scientific and Technical Services	136.1	133.5	107.4	120.2	0.40	0.52
N Administrative and Support Services	90.2	139.5	66.1	99.2	0.42	0.54
R Arts and Recreation Services	76.3	70.3	94.7	79.8	0.36	0.38
S Other Services	45.7	46.9	45.2	46.0	0.44	0.52

Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity.

ble, provided that one knows the appropriate magnification factor.

A Longer Period, 1994-2012

Over longer periods of time, the real share of intermediate inputs can change enough that the growth factors of primary inputs and all inputs can begin to diverge significantly so that the general rule of the magnification factor being approximately equal to the cost share of primary inputs in the base year is less useful. Table 4 presents the same results as Table 2 (using a Laspeyres framework) over the 1994-2012 period.[16] One can see that the errors arising from the primary cost share approximation are often sizable, as large as 38 per cent in wholesale trade and 27 per cent in rental, leasing, and real estate services, reflecting the fact that real primary inputs grew much more slowly than intermediate inputs in the former sector (32 per cent compared to 138 per cent) and much more quickly in the latter (180 per cent compared to 48 per cent). This generally suggests that over longer periods of time, one should be more cautious in using the cost share of primary inputs in the initial year as an approximation of the magnification factor (the real share of primary inputs in the end year). The change in the share of intermediate inputs is more likely to be relevant over longer time periods.

16 The underlying data are presented in Table 3, which is analogous to Table 1.

Table 4

Magnification Factors, Gross Output-Based TFP Relative to Value Added-Based TFP in Australian Industries, Laspeyres Index Number Framework, 1994-2012

	TFP Growth Rate		Magnification Factor	Components of Magnification Factor		
Industry	Gross Output-Based (%) (1)	Value Added-Based(%) (2)	Ratio of Gross Output-based to Value Added-based TFP Growth Rates (3)=(1)/(2)=(4)*(5)	Cost Share of Primary Inputs in Total Input Costs 2011 (4)	Ratio of Growth Factor of Primary Inputs to Growth Factor of All Inputs (5)	Error of Primary Cost Share Approximation (%) (6) = 100*((4)/(3)-1)
A Agriculture, Forestry and Fishing	28.1	90.0	0.313	0.36	0.87	15.2
B Mining	-30.2	-41.0	0.737	0.56	1.32	-24.0
C Manufacturing	2.1	7.3	0.283	0.32	0.88	13.2
D Electricity, Gas, Water and Waste Services	-23.6	34.2	0.691	0.58	1.19	-16.1
E Construction	10.1	38.8	0.259	0.31	0.84	19.7
F Wholesale Trade	16.4	42.6	0.384	0.53	0.72	37.9
G Retail Trade	20.0	39.7	0.503	0.45	1.12	-10.5
H Accommodation and Food Services	7.0	19.3	0.365	0.41	0.89	12.3
I Transport, Postal and Warehousing	9.6	24.1	0.400	0.40	1.00	-0.1
J Information, Media and Telecommunications	-0.4	-1.1	0.386	0.42	0.92	8.8
K Financial and Insurance Services	24.2	44.7	0.542	0.62	0.87	14.4
L Rental, Hiring and Real Estate Services	-24.9	-43.0	0.578	0.42	1.38	-27.4
M Professional, Scientific and Technical Services	5.9	15.8	0.372	0.40	0.93	7.6
N Administrative and Support Services	-8.9	-26.5	0.334	0.42	0.80	25.6
R Arts and Recreation Services	1.6	-4.0	0.391	0.36	1.09	-8.0
S Other Services	-0.4	-0.8	0.437	0.44	0.99	0.7

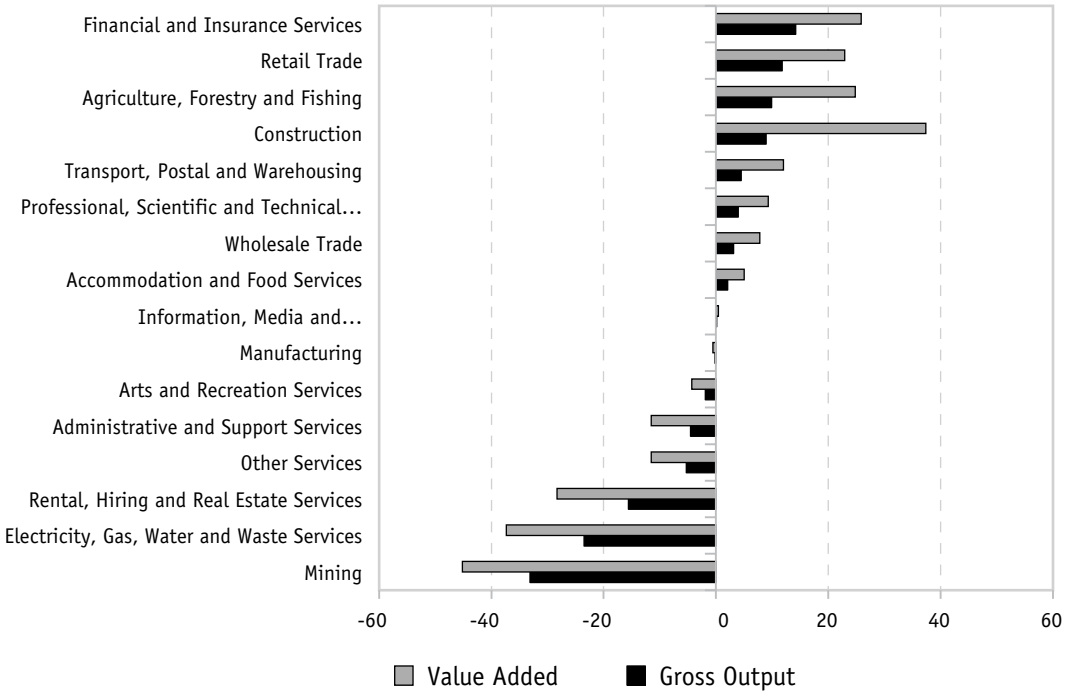
Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity.

TFP Growth Comparisons and the Choice of a Value Added or Gross Output Measure

We have seen that the choice of value added and gross output can lead to significantly different growth rates of TFP (Chart 2), with the absolute value of TFP growth in a value added framework always greater than TFP growth in a gross output framework. This relationship is useful as a reference point when comparing the results of studies which use the two different output measures. However, policymakers are often more interested in the relative productivity performance of industries rather than the absolute level of productivity growth. If TFP growth in the manufacturing sector in a country is 3 per cent in a given year, is this good? Such questions are usually considered by making comparisons to the TFP growth of:

1. Other sectors within the country over the same period, especially the average performance across all industries;
2. The historical performance of the sector of interest within the country;
3. The performance of the same industry in other countries over the same time period.

Chart 2
TFP Growth by Industry,Australia, 2000-2012
(per cent)



Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity. Estimates produced using a Laspeyres framework.

Since relative TFP performance is often of greater interest than the absolute level of growth, the choice of a gross output or value added metric may not be all that important provided that magnification factors are reasonably similar across industries, space, and time.[17]

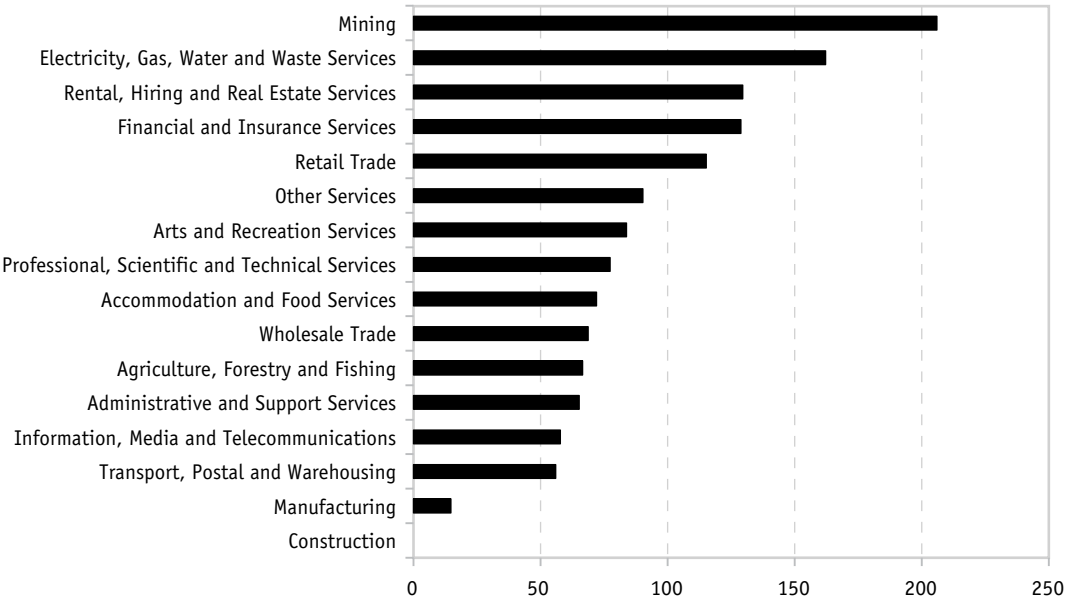
However, we have already seen that there is considerable variation across industries in terms of magnification factors, reflecting differences in the relative importance of intermediate goods in the production process. This can translate into sizable discrepancies between value added-based and gross output-based estimates of TFP growth relative to that of the construction sector. Chart 3 and Table 5 presents the magnification factor of each Australian industry as a per cent deviation from the (approximate) magnification factor of the construction industry for TFP growth between 2000 and 2012.[18] These figures indi-

17 In particular, the relative performance of TFP growth in industry i of country a at time t compared to TFP growth in industry j of country b at time s under a gross output framework is realted to the relative performance under a value added framework by the ratio of the two magnification factors:

$$\frac{\left(\frac{\pi_{G,i,t,a}}{\pi_{G,j,s,b}}\right)}{\left(\frac{\pi_{VA,i,t,a}}{\pi_{VA,j,s,b}}\right)} = \frac{\left(\frac{\pi_{G,i,t,a}}{\pi_{VA,i,t,a}}\right)}{\left(\frac{\pi_{G,j,s,b}}{\pi_{VA,j,s,b}}\right)} = \frac{\left(\frac{1}{\gamma_{i,t,a}}\right)}{\left(\frac{1}{\gamma_{j,s,b}}\right)}$$

18 Industry magnification factors calculated exactly using Laspeyres index number formulae, rather than using estimates of gross output-based TFP and value added-based TFP from the Australian Bureau of Statistics. The construction sector represents a somewhat extreme example in that it has an unusually high share of intermediate inputs in total input costs.

Chart 3
Deviation of Industry Magnification Factors From That of Construction, Australia, 2000-2012
(per cent)



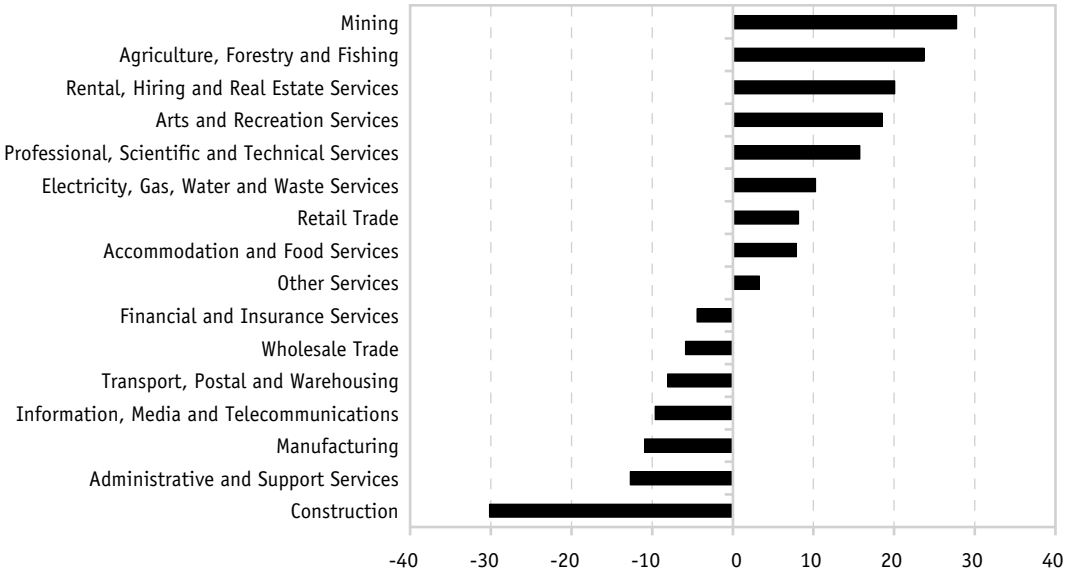
Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity. Estimates produced using a Laspeyres framework.

cate how much greater an industry's deviation from the performance of the construction industry will be under a gross output framework. In some cases, differences relative to the construction sector are increased significantly. For example, growth in financial and insurance services relative to that of the construction industry would appear 129 per cent greater if gross output were used rather than value added for the TFP estimates. This means that if value added-based TFP grew 1.4 times as quickly in financial and insurance services than it did in the overall economy using value added, than it would only grow 0.6 times as quickly using gross output – the choice of output measure would actually change which of the two industries has the higher TFP growth (see Chart 2).

Since construction had the lowest magnification factor over the 2000-2012 period (0.24), reflecting its low intermediate input share, the absolute value of its TFP growth rate relative to that of any other industry with positive TFP growth will always be lower under a gross output framework. Importantly, this does not mean that an industry with a relatively high magnification factor will necessarily appear to have better relative TFP performance when compared under a gross output framework. If construction had experienced negative value added-based TFP growth and was compared to another industry which had negative value added-based TFP growth, then construction would appear to perform relatively well when using a gross output-based TFP measure.

Chart 4 highlights that historical comparisons of TFP growth within industry can also be significantly different under value added and gross output frameworks. Suppose that we

Chart 4
Deviation of 2000-2012 Magnification Factors from the 1994-2000 Magnification Factors Within Australian Industries
(per cent)



Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity. Estimates produced using a Laspeyres framework.

wanted to compare the growth rates of TFP in each industry between 2000 and 2014 to those between 1994 and 2000. The results can vary with the choice of output measure by as much as 30 per cent in some industries such as mining, agriculture, forestry, fishing and hunting, or construction.

Although we do not explore the issue here, the impact of choosing a value added-based or gross output-based TFP measure for comparisons of industries across countries could also be assessed. This could easily be done using the EU KLEMS database, for example.

Conclusion

The decision to employ a gross output or value added measure of output when calculating TFP growth can have a significant effect on the results. In particular, using value added tends to amplify the absolute value of the TFP estimates, resulting in estimates which are between 2 to 3 times as large for most industries. Productivity researchers must be clear as to whether their TFP estimates are based on value added or gross output, and policymakers must be careful to note the distinction when comparing results from different studies. When assessing relative performance across industries or time, it is important to keep the choice of TFP measure in mind. In principle, this could be an issue for comparison of productivity across regions or countries as well, although we leave that as an empirical issue for future exploration.

Over short time horizons, the share of primary inputs in total input use provides a very close approximation to the magnification factor relating gross output and value added TFP measures, so it can serve as a simple way to evaluate the sensitivity of results to the choice

Table 5

Comparison of Relative TFP Growth, Gross Output-Based TFP vs Value Added-Based TFP, Australia, 1994-2000 and 2000-2012

	Magnification Factors		Comparing Industry to Construction Sector (%)		Comparing Industry in 2000-2012 to Itself in 1994-2000 (%)	
Industry	1994-2000	2000-2012	1994-2000	2000-2012	Absolute	Relative to Construction Sector
A Agriculture, Forestry and Fishing	0.32	0.40	0.94	1.66	1.24	1.77
B Mining	0.57	0.73	1.67	3.06	1.28	1.83
C Manufacturing	0.31	0.27	0.90	1.15	0.89	1.28
D Electricity, Gas, Water and Waste Services	0.57	0.63	1.66	2.62	1.10	1.58
E Construction	0.34	0.24	1.00	1.00	0.70	1.00
F Wholesale Trade	0.43	0.40	1.25	1.69	0.94	1.35
G Retail Trade	0.48	0.52	1.39	2.15	1.08	1.55
H Accommodation and Food Services	0.38	0.41	1.11	1.72	1.08	1.54
I Transport, Postal and Warehousing	0.41	0.37	1.19	1.56	0.92	1.32
J Information, Media and Telecommunications	0.42	0.38	1.22	1.58	0.90	1.29
K Financial and Insurance Services	0.57	0.55	1.67	2.29	0.96	1.37
L Rental, Hiring and Real Estate Services	0.46	0.55	1.34	2.30	1.20	1.72
M Professional, Scientific and Technical Services	0.37	0.42	1.07	1.77	1.16	1.66
N Administrative and Support Services	0.45	0.40	1.32	1.65	0.87	1.25
R Arts and Recreation Services	0.37	0.44	1.08	1.84	1.19	1.70
S Other Services	0.44	0.46	1.29	1.90	1.03	1.48
Market Sector Industries	0.41	0.44	1.00	1.00	1.07	1.00

Source: Author's calculations using data from the Australian Bureau of Statistics Estimates of Industry Multifactor Productivity

of TFP measure or to compare studies which use different output measures. However, over longer time horizons we find that this approximation is less reliable because there can be significant changes in the share of intermediate inputs in total input costs, implying that the change in the real share of intermediate inputs should be considered when evaluating the relationship between value added-based and gross output-based TFP growth.

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Appendix
The Relationship between Gross Output-based TFP and Value Added-based TFP Growth: Paasche and Fisher Frameworks

SIMILAR TO THE LASPEYRES CASE, Diewert (2015, equation 28) derives an exact expression under a Paasche framework (equation 28). The magnification factor in this case is:

$$\frac{1}{\overset{*}{\gamma}} = \frac{\overset{*}{\pi}_G}{\overset{*}{\pi}_{VA}} = \frac{(1-s_M^1)\Gamma_Y}{[S_Y^1\Gamma_Y^{-1} + (1-S_M^1)\Gamma_M^{-1}]^{-1}}$$

where an asterisk (*) denotes that the variable corresponds to a Paasche index framework, a superscript 1 indicates year 1, Γ_Y is the growth in gross output, $S_Y^1 > 0$ is the time 1 value added expenditure "share" of gross output, and $S_M^1 < 0$ is the time 1 value added expenditure "share" of intermediate inputs.[19] Similar to the Laspeyres magnification factor, this can be split into the time 1 share of primary input, $(1-s_M^1)$, and a ratio of the growth factor of gross output to that of real value added.[20] Since the latter term is approximately equal to one provided that the growth rates are similar or small, the Paasche magnification factor is:

$$\frac{\overset{*}{\pi}_G}{\overset{*}{\pi}_{VA}} \approx (1-s_M^1)$$

These expressions for the Paasche and Laspeyres magnification factors are exact. Fisher indexes, which are a geometric average of Paasche and Laspeyres indexes, are popular because they have many desirable properties. The Paasche and Laspeyres magnification factors can be used to approximate the magnification factor in a Fisher framework as (Diewert 2015, equation 39):

$$\frac{1}{\overset{F}{\gamma}} = \frac{\overset{F}{\pi}_G}{\overset{F}{\pi}_{VA}} \approx \frac{1}{w\gamma + (1-w)\overset{*}{\gamma}}$$

where the weight given to the Laspeyres magnification factor is defined using the Paasche and Laspeyres gross output-based TFP growth rates as $w \equiv \frac{\pi_G}{\pi_G + \overset{*}{\pi}_G}$.

We will refer to this as the "complex" approximation of the Fisher magnification factor. Under the assumption that the Paasche and Laspeyres growth rates are very similar, Diewert (2015) offers a simpler approximation where $w = 1/2$.

Appendix Table 2 of the on-line appendix presents the magnification factors calculated using TFP growth in a Fisher index framework alongside the "simple" and "complex" approximations over a one-year time horizon. Both approximations tend to be very accurate. The errors when using the complex approximation are negligible, but in most cases there is not much gained from

19 These real value added output expenditure shares represent the "contributions" of gross output and intermediates to value added, as the nominal value of value added in year 1 is equal to the nominal value of gross output minus the nominal value of intermediate inputs. Note that these "shares" sum to 1, but S_Y^1 will be greater than one while S_M^1 will be negative because the contribution of intermediates is negative. Diewert (2015) shows that these are related to the share of primaries in total expenditures by the expressions $S_Y^1 = \frac{1}{s_X^1}$ and $S_M^1 = \frac{-(1-s_X^1)}{s_X^1}$.

20 Notice that

$$(1-s_M^1) = \frac{\textit{Value of Value Added}_1}{\textit{Value of Gross Output}_1} = \frac{\textit{RealValue Added}_0}{\textit{Real Gross Output}_0} = \frac{[S_Y^1\Gamma_Y^{-1} + (1-S_M^1)\Gamma_M^{-1}]^{-1}}{\Gamma_Y}$$

so that the Paasche magnification factor can also be interpreted as simply the ratio of real value added to real gross output at time 0.

calculating the appropriate weights for the Paasche and Laspeyres magnification factors. Over a one year period, the assumption that the Paasche and Laspeyres TFP growth rates are about the same is quite reasonable. Mining stands out as the only industry where the simple approximation has a notable error, which is related to large annual fluctuations in the relative price of the output of this sector, but even this is only about 1 per cent.

In most industries, the choice of a Laspeyres, Paasche, or Fisher index for aggregation has very little impact on the estimated magnification factor (Appendix Table 3). Mining is somewhat of an exception, with the magnification factor ranging from 0.5 under a Paasche framework to 0.6 under a Laspeyres framework. This is likely attributable to the large price swings in natural resources in the 2011-2012 period. For comparison, the magnification factors based on the official ABS estimates of TFP growth are also included in Appendix Table 3. They tend to be in line with the other magnification factors that we have generated using the ABS data. This suggests that the share of primary costs can be a reasonably good estimator of the magnification factor over short time periods for TFP estimates produced using a combination of Laspeyres and Tornqvist-Theil indexes as well.

The approximations for the magnification factors under a Fisher framework also are less accurate over longer time horizons as they do over short periods, although they still tend to be reasonably good. While the complex approximation becomes somewhat less accurate, most of the errors remain very small – notable exceptions include mining (about 1 per cent), information, media, and telecommunications (about 6 per cent), and administrative and support services (about 4 per cent). In the majority of cases, the simple approximation still performs very well, but there can sometimes be sizable errors. The most striking example is information, media, and telecommunications where the simple approximation leads to an estimate 64 per cent lower than the actual magnification factor. It is not difficult to see that the problem in this case relates to an unusual situation where the weight is negative. This arises because the Paasche growth rate is positive while the Laspeyres growth rate is negative. The simple approximation also leads to a large error of 27 per cent in administrative and support services. Consequently, it should be applied cautiously over long time horizons.

Appendix Table 5 compares the magnification factors under the three index number approaches to those from the official ABS estimates which relied on chained Tornqvist-Theil and chained Laspeyres indexes. Over long time horizons, there are opportunities for large changes in prices so that the choice of index approach can have a significant effect on the results. While the cost shares of primary inputs tend to be reasonably stable over time, some of them have changed considerably between 1994 and 2012. For example, the cost share of primary inputs rose 31 per cent in administrative and support services from 0.42 to 0.55 per cent. Such an increase can have a substantial impact on the magnification factor, leading to significant differences between the Paasche (magnification factor of 0.64) and Laspeyres (0.334) index approaches. In other cases, there is broad consistency between estimates using all three index number approaches and the official data – agriculture, forestry, and fishing or transport, postal, and warehousing, for example.